Beam Induced Damage Mechanisms and Their Calculation

Alessandro Bertarelli

with contributions from
F. Carra, A. Dallocchio, M. Garlaschè, P. Gradassi

CERN, Geneva, Switzerland

Outline

- Objective and Scope of the Lectures
- Part I: Introduction to Beam-induced Accidents
- Part II: Analysis of Beam Interaction with Matter
- Part III: Design Principles of Beam Interacting Devices
- Part IV: Experimental Testing and Validation

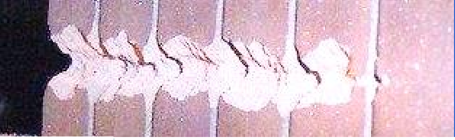
Objectives and Scope of the Lectures

- We deal with **rapid** and **intense interactions** between **particle beams** and **accelerator components** (typically lasting **ns** to **μs**). We do not treat other energy release mechanisms (e.g. of *stored magnetic energy or RF impedance-induced heating*)
- Cover **damage mechanisms** occurring in the **μs to few seconds time scale**. Longer term phenomena (e.g. *radiation damage*) not extensively covered (see **N. Mokhov**'s lecture)
- Specific focus on **high energy**, **high intensity accelerators** where these events are more dangerous (although this can be extended to any particle accelerator …).
- Mainly cover components directly exposed to interaction with beam (**Beam Interacting Devices,** e.g. *targets, dumps, absorbers, collimators, scrapers, windows …*)
- However, **mechanisms extend to any other component** accidentally and rapidly interacting with energetic beams (*vacuum chambers, magnets, RF cavities, beam instrumentation*).
- Analysis mostly deal with **isotropic materials**. Principles can be extended to **anistropic** materials with some mathematical complexity.

Objectives and Scope of the Lectures

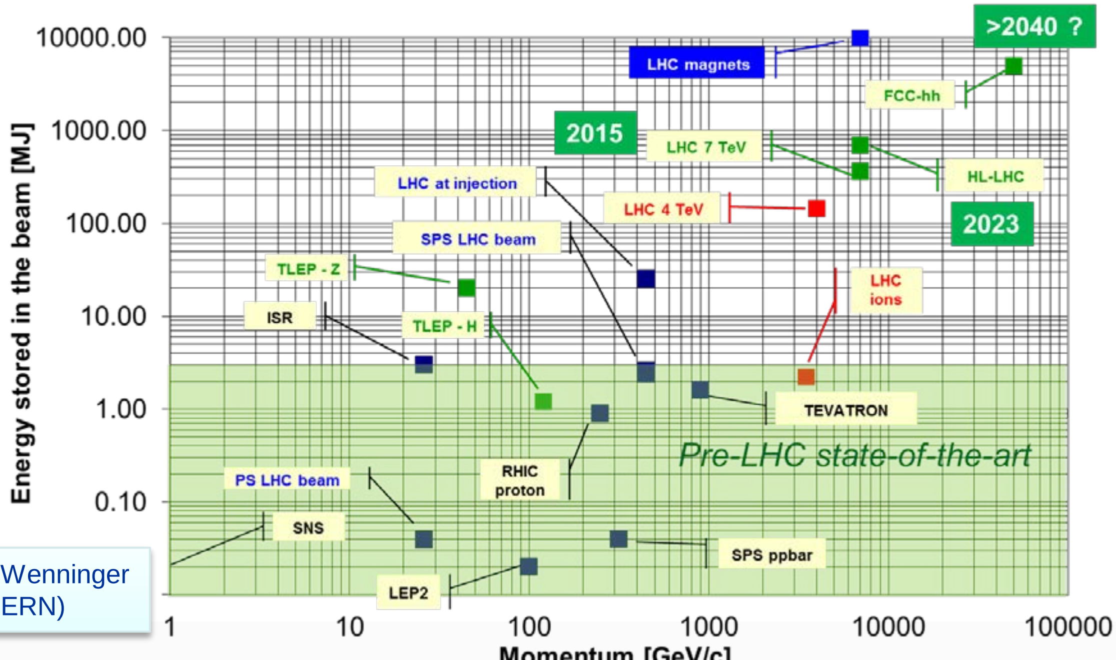
- In **first lecture**, focus is given on the theoretical and thermo-mechanical principles allowing to **analyze the phenomena** from an engineering perspective.
- In **second lecture**, we deal with the **design of beam interacting systems** treating aspects as figures of merit, intensity limits, advanced materials, testing facilities etc.

- Objective and scope of the lectures
- Part I: Introduction to Beam-induced Damage
  - High Energy Particle Accelerator Challenges
  - A Gallery of Beam-induced Accidents
  - Multiphysics Approach to Beam-induced Damages
- Part II: Analysis of Beam-Matter Interaction
- Part III: Design Principles of Beam-interacting Systems
- Part IV: Experimental Testing and Validation



High Energy Particle Accelerators Challenges

- Beams circulating in last-generation accelerators can store **very high energies** (**678 MJ** for future **HL-LHC**)
- What matters is not only the energy content, but also **how short it takes to release** it!
- The energy of 1 HL-LHC beam can potentially be unleashed in a **few tens of microseconds**!
- No surprise, **beam-induced accidents** represent one of the **most dangerous** and though **less explored** events for Particle Accelerators!



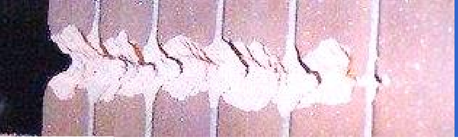
J. Wenninger (CERN)

What is HL-LHC Energy equivalent to?

USS Harry S. Truman

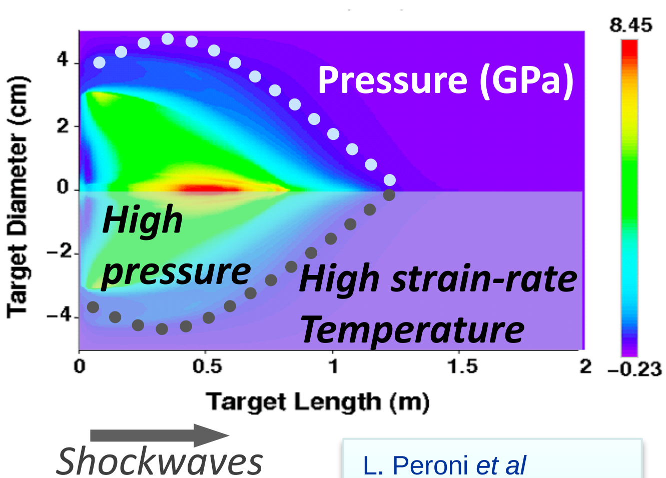
TGV

160 kg TNT

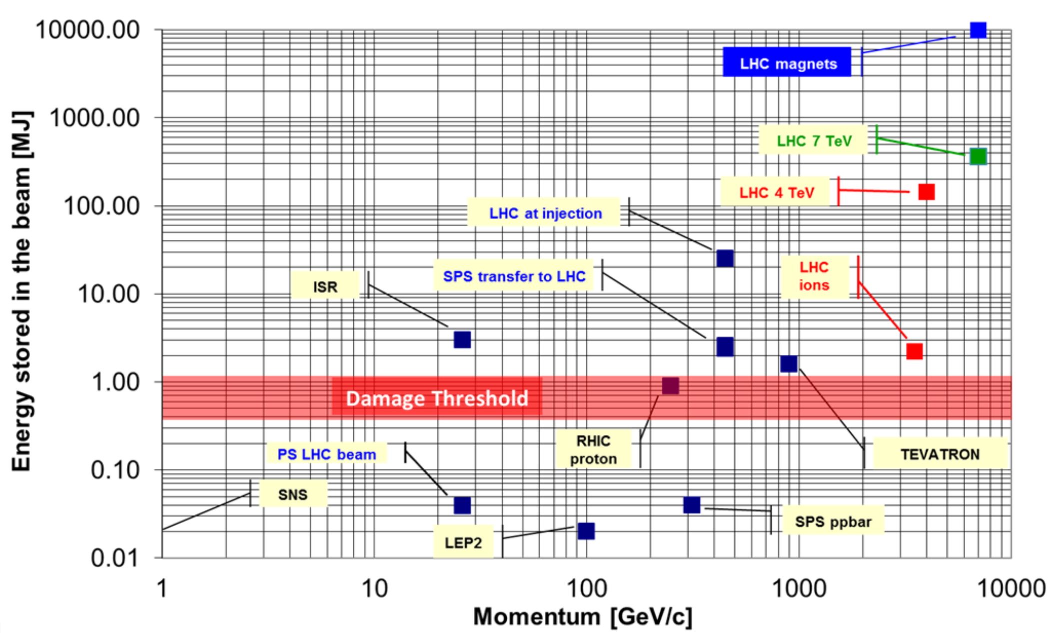


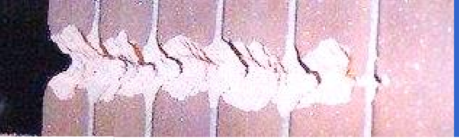
Particle Accelerator Challenges

- **Rapid interaction** of highly energetic beams with matter leads to a number of phenomena:
  - Sudden **temperature rise** of the impacted component. Where energy deposition is more intense **changes of phase** can occur (melting, vaporization, plasma ….)
  - Regions not-undergoing phase transitions are anyway subjected to heating and high thermal deformations in very short time (**high strain-rate**), with propagation of **intense pressure waves** possibly leading to extended **mechanical damage**.



L. Peroni *et al*
(Politecnico di Torino)





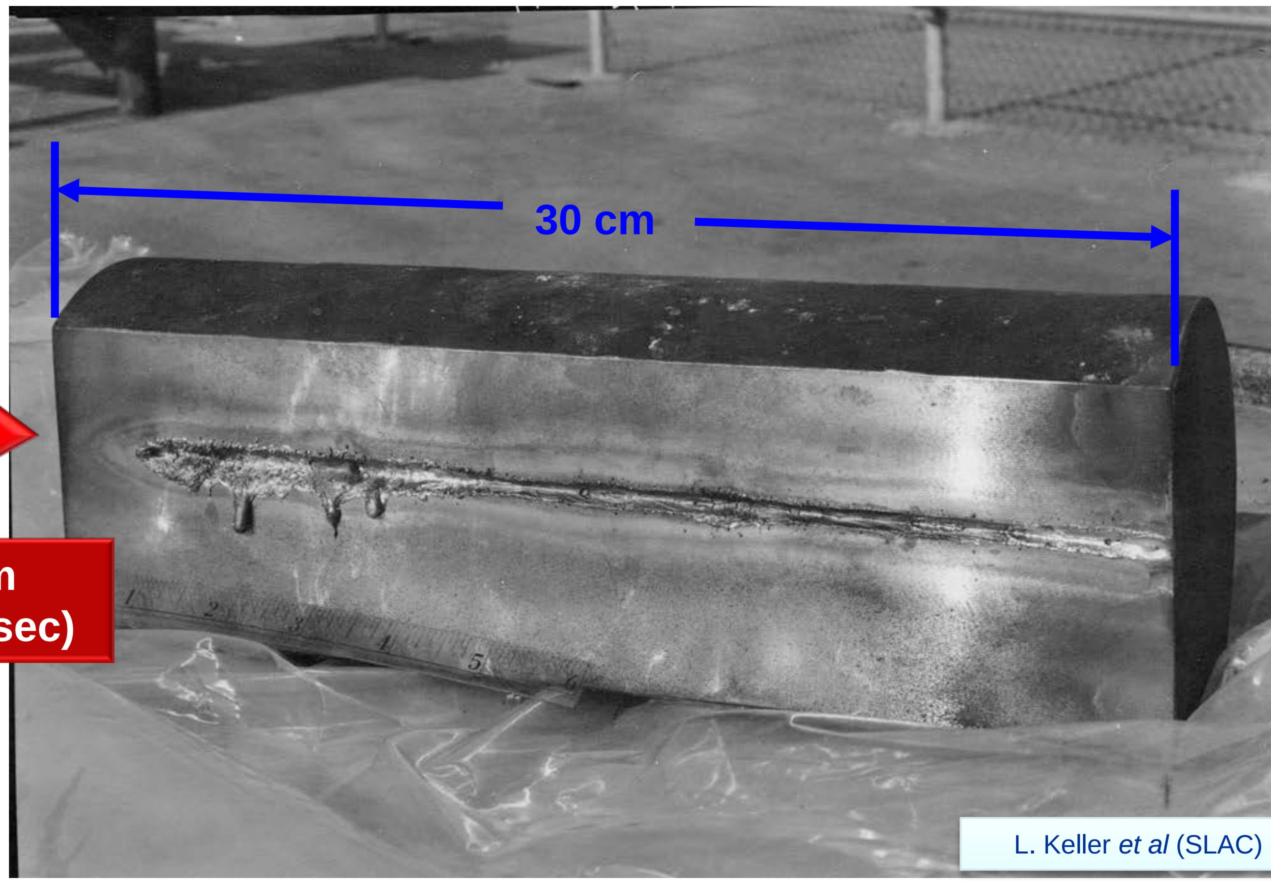
Permanent Deformation of SPS Target Rod

- First neutrino target (**Beryllium**, 3 mm diameter, 100 mm long) at CERN-SPS impacted by a 300 GeV, 1×10^{13} protons, pulse duration 23 μs. **Early 70's**.
- Target permanently bent …
- **Q**: Any idea of the reason?

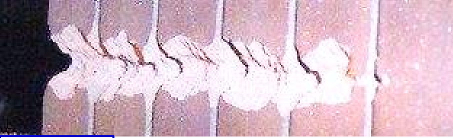
P. Sievers *et al* (CERN)

SLAC Damage Test on Cu Block

- Damage test of a 30 cm long **Copper** Block (SLAC – **1971**)
- A ~2-mm 500 kW Beam enters a few mm from the edge.
- It took about 1.3 sec to melt through the block (slow accident).

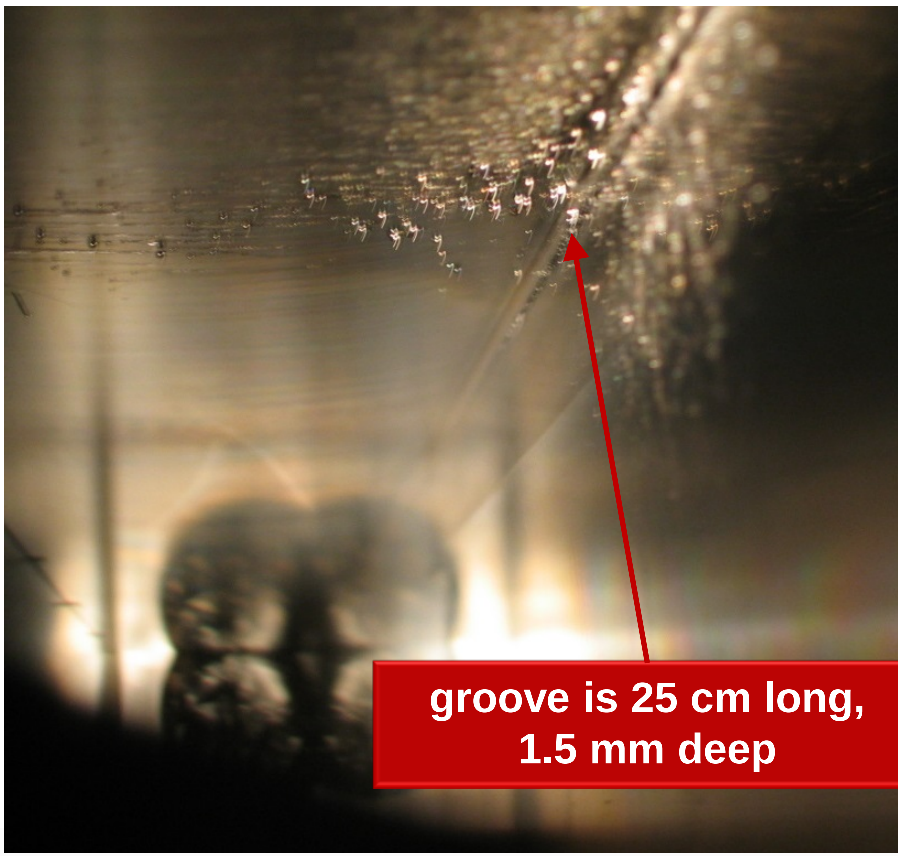


L. Keller *et al* (SLAC)



Tevatron Collimator Accident

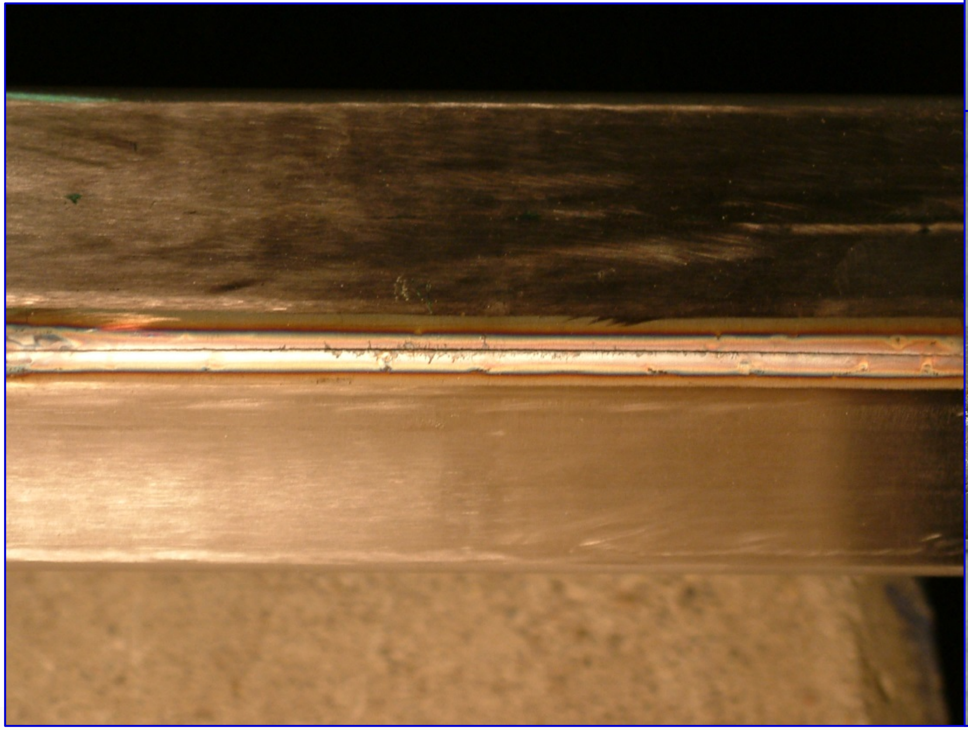
- In **2003** a Roman pot (movable device) accidentally moved into the Tevatron beam.
- Beam moved by 0.005 mm/turn, and touched primary (**Tungsten**) and secondary (**Stainless Steel**) collimator jaws surface after about 300 turns
- The entire beam was lost, mostly on collimators (~0.5 MJ)

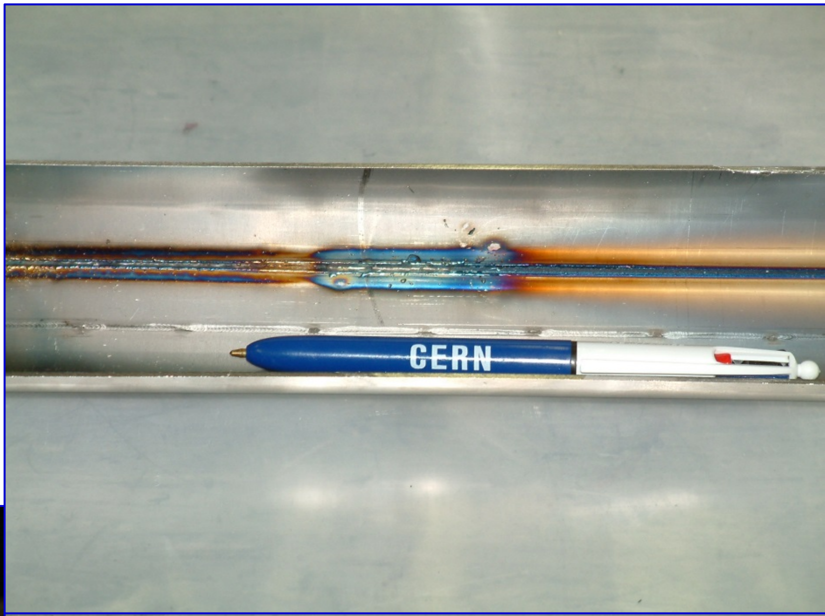


N. Mokhov *et al* (FNAL)

SPS Extraction Line Accident

- 450 GeV protons, 2 MJ beam in **2004**
- Failure of a septum magnet induced beam drift
- Cut of 25 cm length, groove of 70 cm on **Stainless Steel vacuum chamber**
- Condensed drops of steel on opposite side of the vacuum chamber
- Vacuum chamber and magnet to be replaced



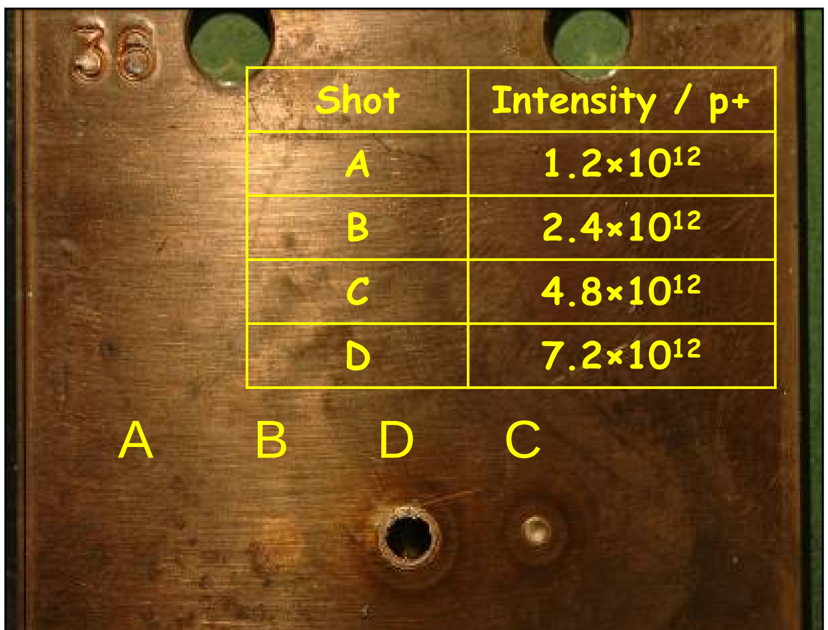


B. Goddard, R. Schmidt *et al* (CERN)

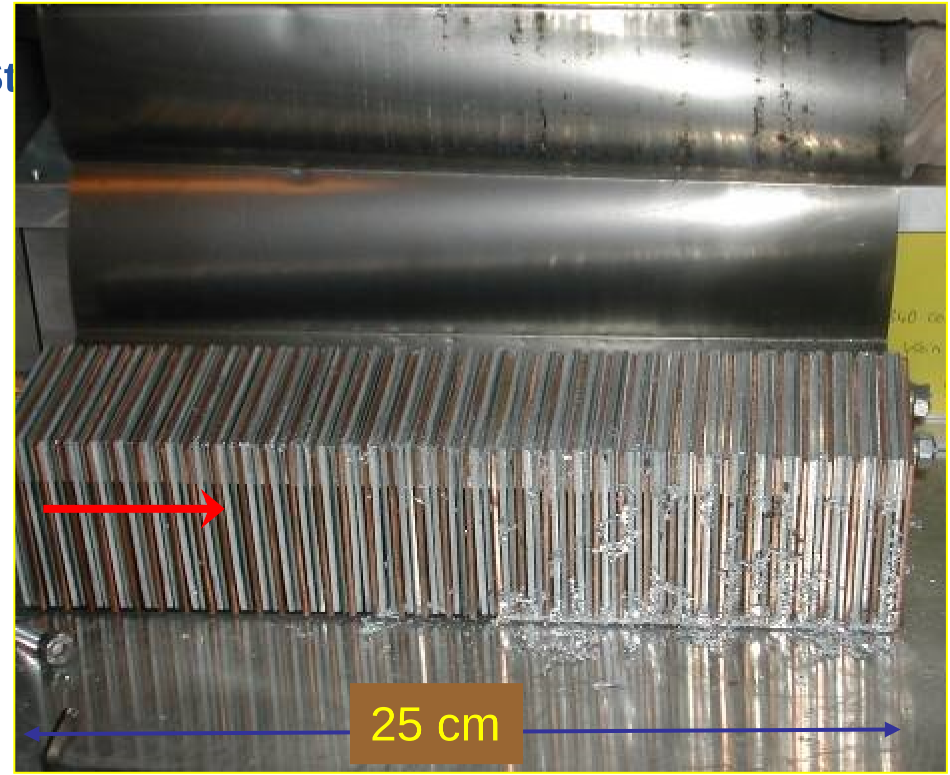
SPS Material Damage Experiment

Controlled SPS experiment (450 GeV) in **2004** on stack of **Copper** and **Stainless St** plates

- Up to ~$8{\cdot}10^{12}$ protons
- Beam size $\sigma_{x/y}$ = 1.1mm/0.6mm
- Visible effects on Cu above ~ $2{\cdot}10^{12}$ p
- Stainless steel: no visible damage

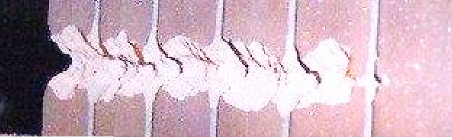


Shot	Intensity / p+
A	$1.2{\times}10^{12}$
B	$2.4{\times}10^{12}$
C	$4.8{\times}10^{12}$
D	$7.2{\times}10^{12}$



- 0.1% of the full LHC 7 TeV beams
- ~One quarter of LHC injection train
- Copper damage limit ~200 kJ

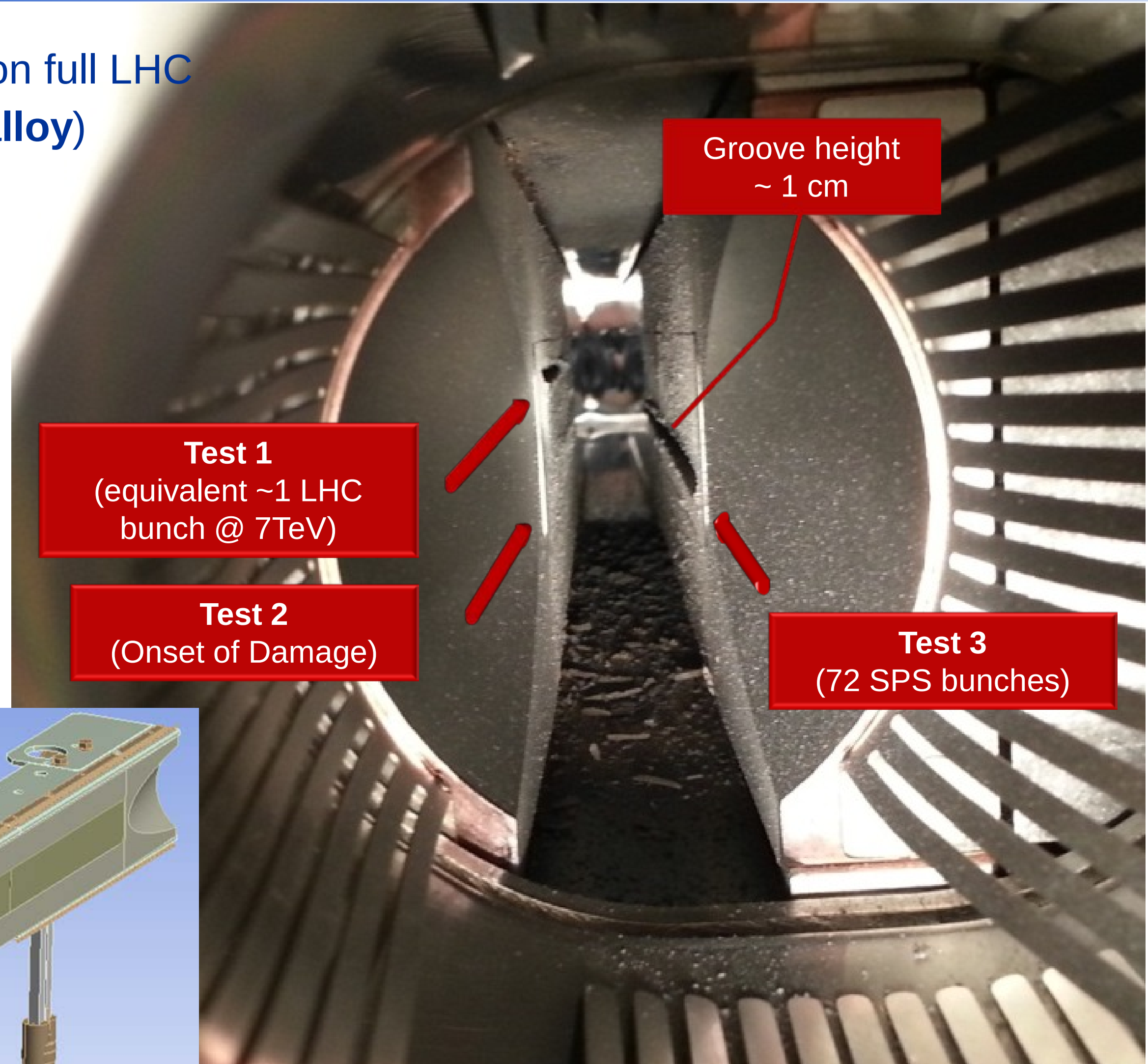
V. Kain, R. Schmidt *et al* (CERN)

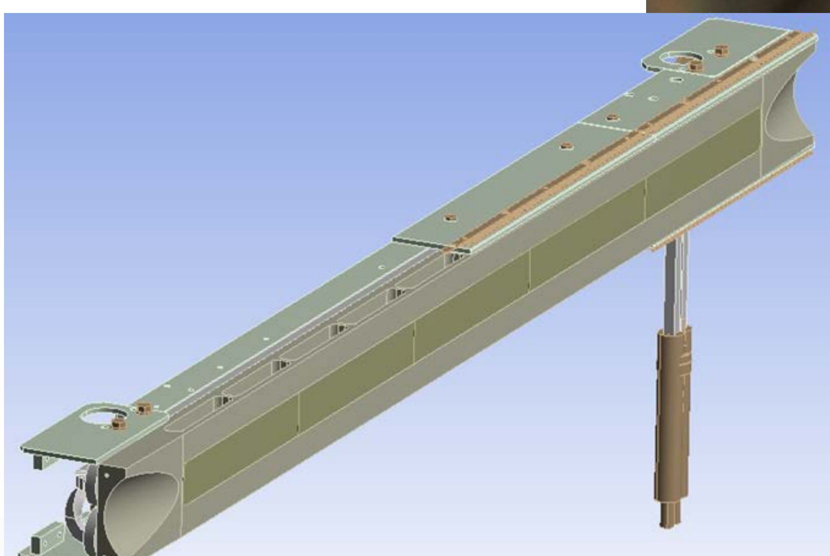


HiRadMat Impact Test on W Collimator

2012 test in HiRadMat Facility on full LHC Tertiary Collimator (**Tungsten alloy**)

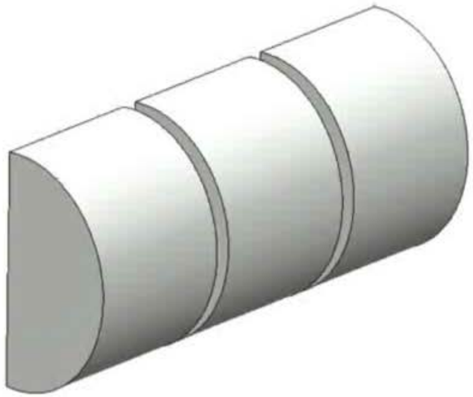
- 450 GeV
- Up to $9.34 \cdot 10^{12}$ protons
- Beam size ($\sigma_{x/y}$) 0.53x0.36 mm^2
- Impact depth: 2 mm
- 3 shots (24, 6, 72 bunches)





HiRadMat Test on Material Sample Holder

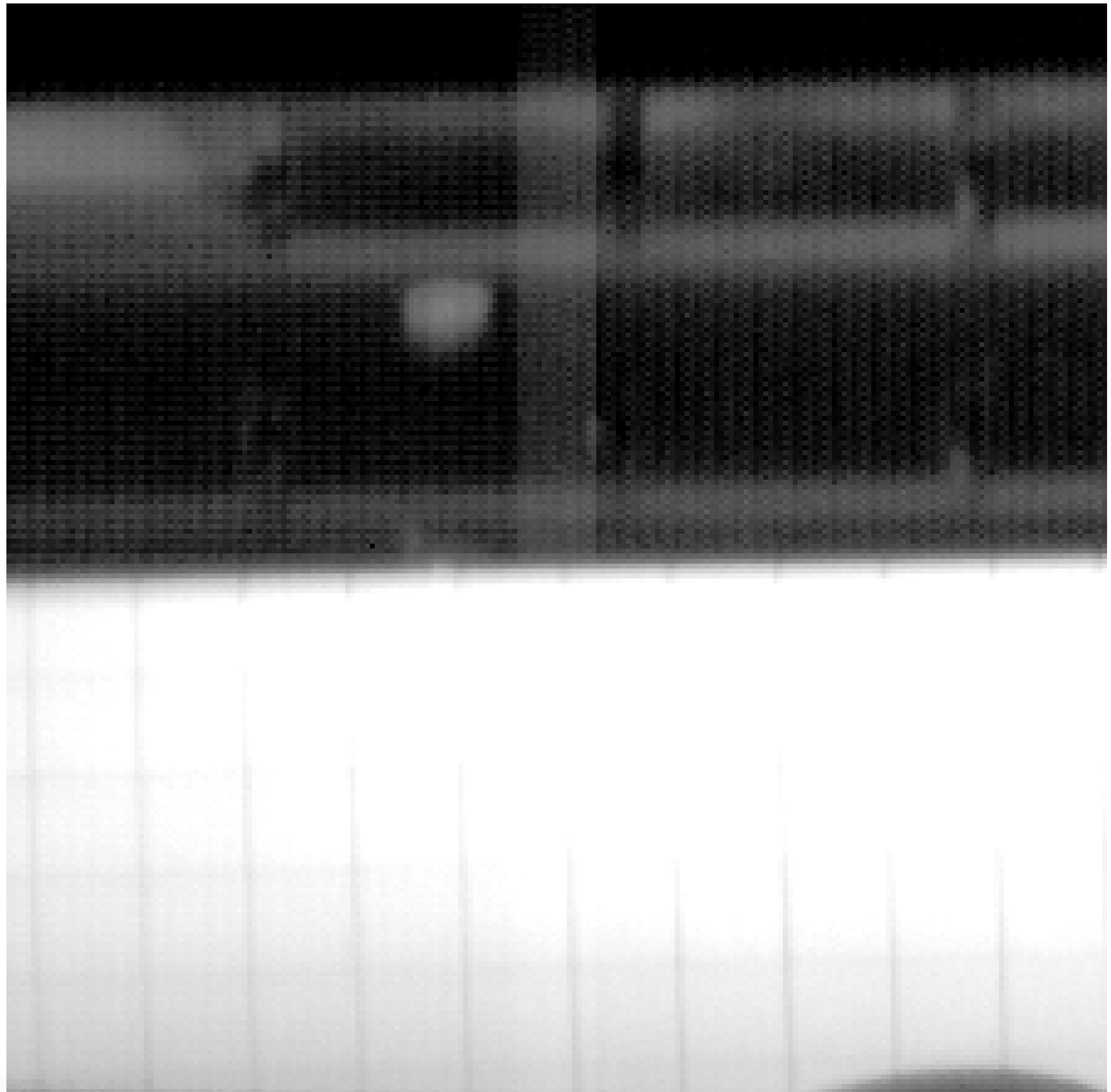
- **2012** test in HiRadMat facility on sample holder hosting specimens made of **6 different materials**.
- Tungsten alloy (**Inermet 180**) specimens inside experiment vacuum vessel as seen from viewport and high-speed camera



HiRadMat Test on Material Sample Holder

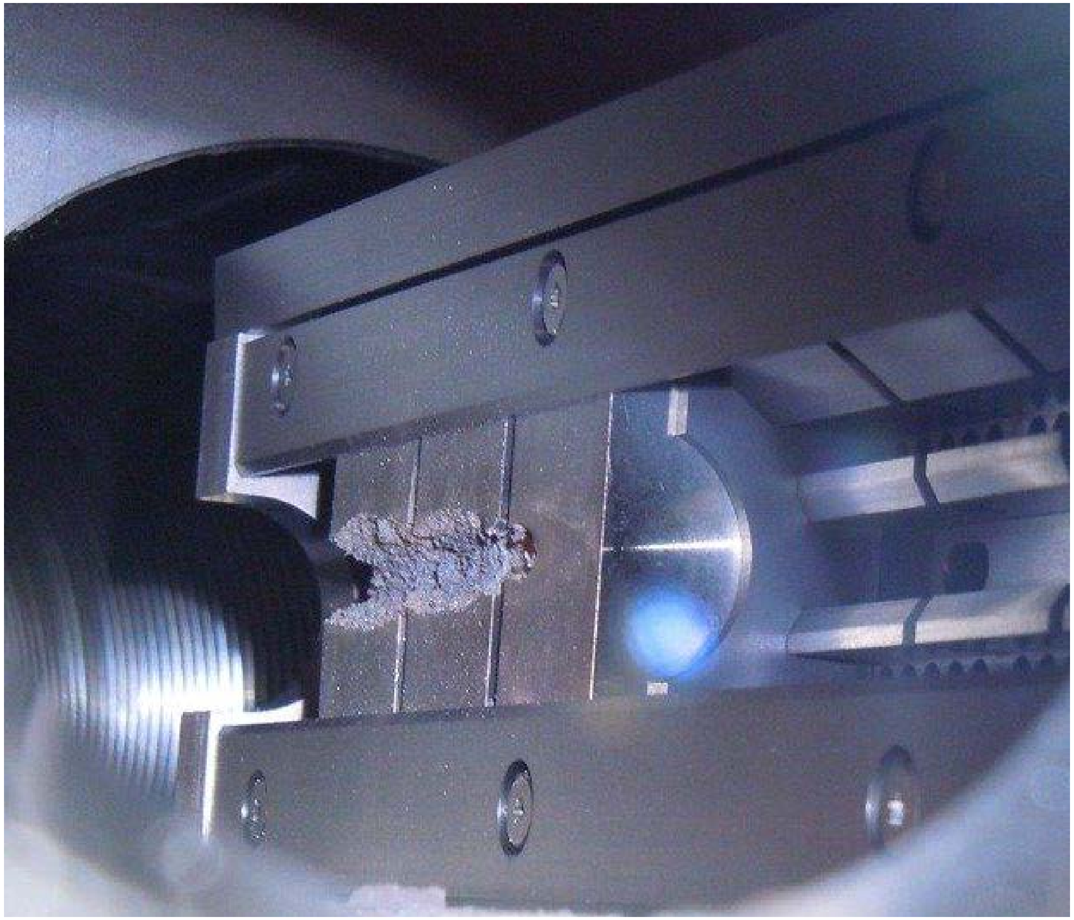
- Impact of 72 b SPS beam on 3 **Inermet 180** specimens

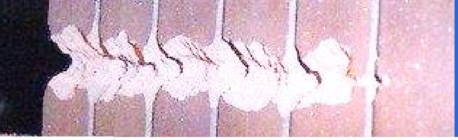
Beam →



HiRadMat Test on Material Sample Holder

- Post-irradiation observations on 3 **Inermet 180** specimens

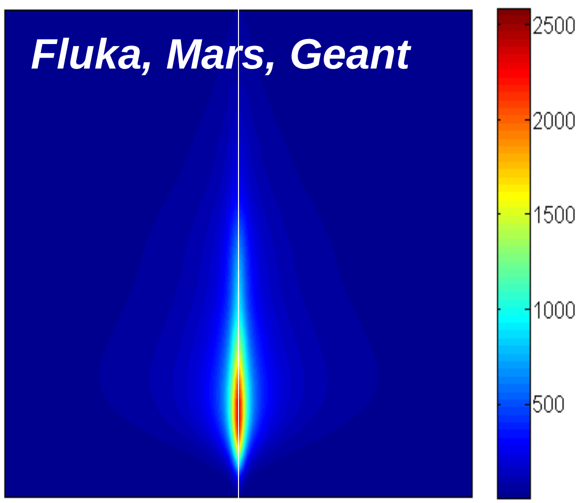




Multiphysics Approach to Beam-induced Damage

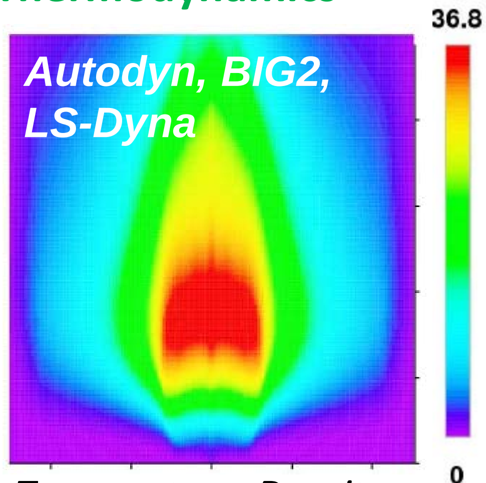
- **Damage phenomena** induced by **high energy**, **high intensity** particle beams bring matter to **extreme regimes** where practical experience and material knowledge is very limited.
- Accurate prediction of structure responses to such events is thus very complex.
- *The analysis of these phenomena must rely on methodologies integrating and coupling several disciplines , numerical tools and experimental verification in a **multiphysics approach**.*

Physics



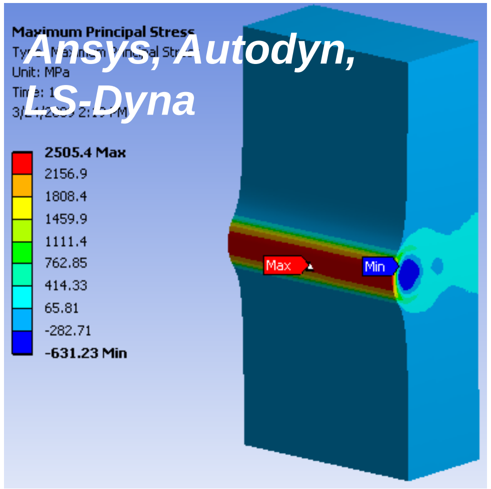
Energy

Thermodynamics



Temperature, Density, Pressure

Structural/mechanical engineering



Strains, Stress, Damage

Complex geometry, complex material behavior, complex boundary conditions …

- Objective and scope of the lectures
- Part I: Introduction to Beam-induced Damage
- Part II: Analysis of Beam Interaction with Matter
  - The Physical Problem
  - The Thermal Problem
  - The Linear Thermomechanical Problem
  - The Non-linear Thermomechanical Problem
- Part III: Design Principles of Beam-interacting Systems
- Part IV: Experimental Testing and Validation

Methodological approach

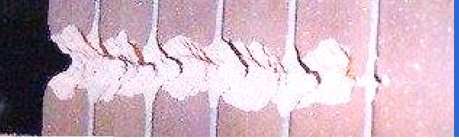
In this part will deal with analysis of the beam-matter interaction problems from an **engineering perspective**.

The analysis can be divided **3 main steps** in sequential order.

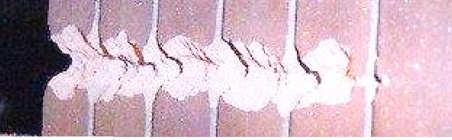
1. The **Physical Problem**. The goal is to determine **how much energy and where** has been deposited onto the body

2. The **Thermal Problem**. The goal is to the determine **which temperature distribution** (at which moment in time) has been induced in the body by the deposited energy.

3. The **Thermomechanical problem** (linear and nonlinear). The goal is to determine **which "forces", deformations, dynamic response and phase transitions** have been generated in the body by the thermal field.

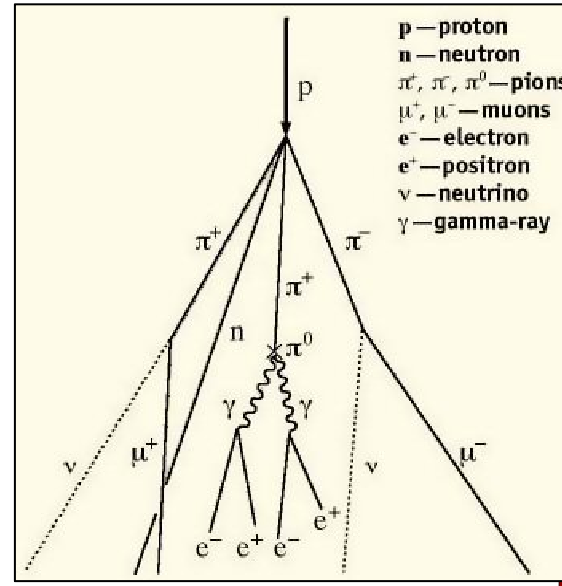


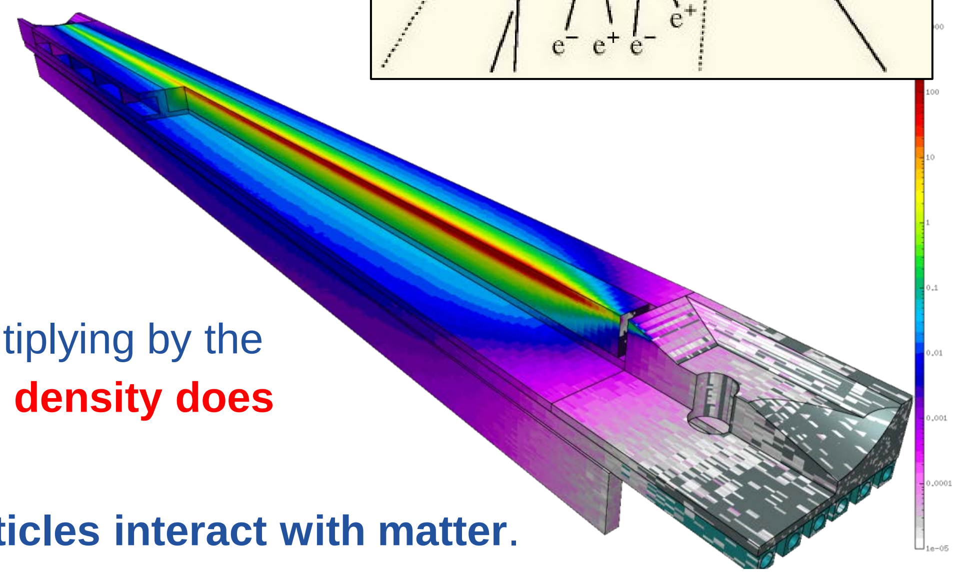
- ✓ Analysis of the Physical Problem
  - ✓ Interaction of Particle Beams with Matter
  - ✓ Energy Deposition and Heat Generation
  - ✓ Duration of Energy Deposition
  - ✓ Temperature Increase
  - ✓ Changes of density



Interaction of Particles with Matter

- **Particles interact with matter through various mechanisms** (depending on particle energy, species, material density, atomic number, etc.)
- Part of each particle **energy is lost** in the target and **ultimately transformed into heat**
- **Monte-Carlo Interaction/Transport codes** are used to simulate these phenomena and derive **energy deposition maps** (see *N. Mokhov and F. Cerutti lectures*)
- Interactions occur at the speed of light ⇨ **each particle deposits heat almost instantaneously**
- **Total deposited energy** is calculated multiplying by the number of interacting particles (**provided density does not change during interaction** …)
- **Energy deposition lasts as long as particles interact with matter**. This depends on bunch length, number of interacting bunches, bunch spacing …





L. Skordis (CERN)

Duration and Power of Beam Impacts

- A particle bunch has a typical (time) length in the order of **1 ns**
- Bunches are typically separated by a **few tens of ns** (**bunch spacing**).

Q: What are duration τ, Energy Q_I and power $\dot{Q}_I$ involved in a beam impact?

Example: **CERN-SPS** extraction train:

bunches $n_b = 288$, *bunch spacing* $t_b = 25\ ns$ ⇨ $\tau = n_b \cdot t_b = 7.2\ \mu s$

Energy $q_p = 450\ GeV$, *p per bunch* $N = 1.3 \cdot 10^{11}\ p/b$ ⇨

Energy of the Impacting Beam $Q_I = n_b \cdot N \cdot q_p = 2.7\ MJ$

Power of the Impacting Beam $\dot{Q}_I = Q_I/\tau = 374\ GW$

Equivalent to ~300 large Nuclear power plants!!

Luckily, it is very short and (usually) not whole energy is deposited!

- **Beam impacts** are typically **very intense and short phenomena** (although longer interactions exist).

- **Homework: Calculate Energy and Power of an LHC full proton beam (e.g. impacting on the LHC beam dump)** [*LHC beam : 2808 b, 25 ns bunch spacing, 1.15e11 p/b 7 TeV*]

- Once energy deposition maps are available, temperature profiles can be calculated **provided the heat diffusion time is much slower than the duration of the impact τ** (**assumption to be carefully verified!**) via the material **specific heat**

$$q_V(x_i) = \int_{T_i}^{T_f} \rho \cdot c_p(T) \cdot dT(x_i) \quad (1)$$

where:

q_V *is the* ***deposited energy*** *per unit of volume [J cm*$^{-3}$*]*

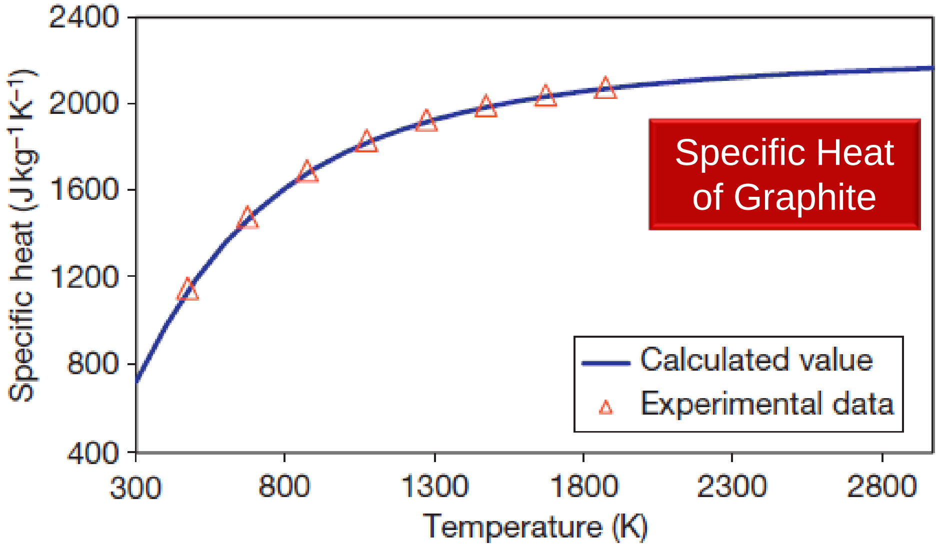
ρ *is the* ***density*** *of impacted material [g cm*$^{-3}$*]*

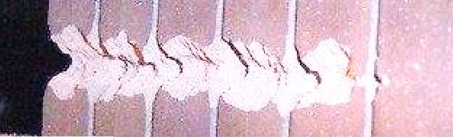
c_p *is the* ***specific heat*** *[J g*$^{-1}$*K*$^{-1}$*]*

T_i *and* T_f *are the* ***initial*** *and* ***final temperatures*** *[K]*

- In general, the c_p is a **function of temperature** (strong dependence in some cases, e.g. *graphite*). However, as a first approximation, an **average value** $\bar{c}_p$ can be taken to get **quasi-instantaneous temperature *increase***

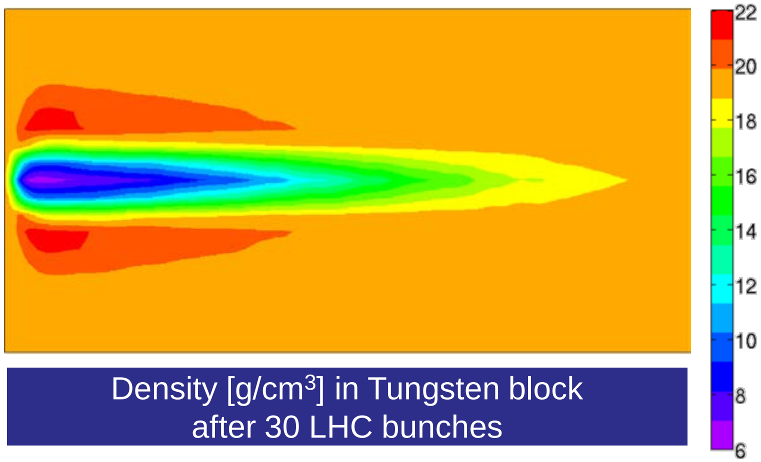
$$\Delta T(x_i) \cong \frac{q_V(x_i)}{\rho \cdot \bar{c}_p} \quad (1a)$$



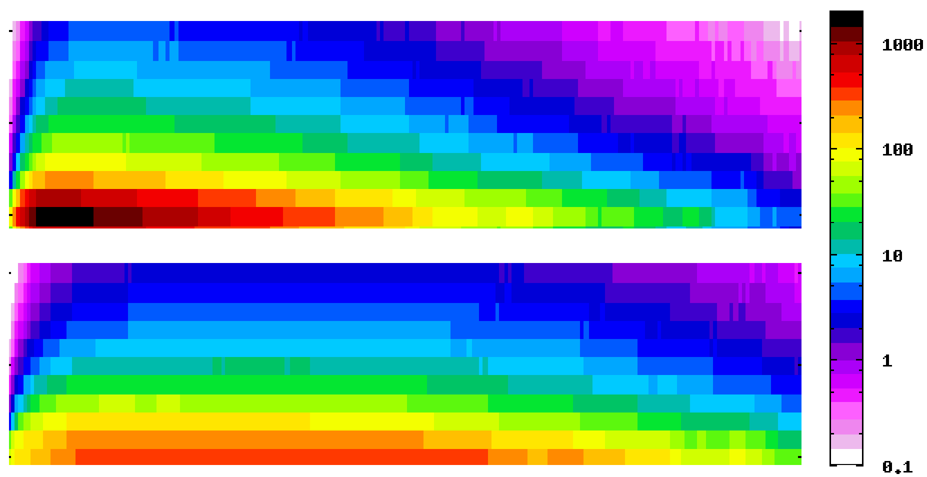


Temperature Increase vs. Change of Density

- Temperature increases can be **very high**, particularly in the regions of peak energy deposition (**several thousands degrees**)
- **Melting temperatures** (particularly of metals) can be exceeded **during** the impact. In this case one or more **phase transitions** occur, usually accompanied by drastic **changes of density**
- In this case, particular caution must be taken since if interaction with beam is still on-going while density varies, the **energy deposition map is modified by the change of density** (*lower energy peaks, deeper penetration* …)



Density [g/cm^3] in Tungsten block after 30 LHC bunches



Energy deposition [GeV/cm^3] on Tungsten block with and without effect of Density change

L. Peroni, M. Scapin (Politecnico di Torino)
V. Boccone (CERN)

- ✓ Analysis of the Thermal Problem
  - ✓ Equation of Heat Diffusion (Fourier's Equation)
  - ✓ Thermal Diffusion Time
  - ✓ Thermal Diffusion Time vs. Impact Duration

- In order to assess the stress state in a body submitted to **thermal shocks**, it is fundamental to determine the **initial temperature distribution** and its **evolution in time**.
- The evolution of temperature is governed by a **diffusion process**, the **Heat Equation (Fourier's Equation)**:

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (\lambda \nabla T) + \dot{q}_V \qquad (2)$$

where $\dot{q}_V$ *is the* ***heat generation rate*** *[W m[-3]]*

λ *is the* ***thermal conductivity*** *[W m[-1]K[-1]]*

- Assuming the body is **homogeneous** and **isotropic** (not always true) we get:

$$\frac{\partial T}{\partial t} = a \nabla^2 T + \frac{\dot{q}_V}{\rho c_p} \qquad (2a)$$

$a = \frac{\lambda}{\rho c_p}$ *is the* ***thermal diffusivity*** *[m[2] s[-1]]*

- If all material properties are constant, (2a) is a PDE with constant coefficients, which, in some cases, can be solved analytically.

Nota Bene*: the* ***Heat Equation fails to predict heat transfer phenomena at very short time scales****, since it implies infinite speed of heat signal propagation. This is usually not relevant in our problems, but can play a role in even shorter phenomena, e.g.,* ***high-frequency laser pulsed heating****.*

Heat Equation: Thermal Diffusion Time

- Once energy deposition is completed, Eq. (2) becomes a **homogeneous linear PDE**
- We assume that **initial temperature distribution is given** (e.g. by Eq. (1a))
- Given the short time scales, we may reasonably consider that the **system adiabatic** (all energy conserved) **regardless of actual boundary conditions** (e.g. *active cooling*)
- Analytical solutions of the Heat Equation are available for simple geometries, usually involving Fourier series, Bessel series, Laplace transforms etc.
  E.g. for a **circular cylinder** (or **disk**) impacted at its center (**axially symmetric load**) we have:

$$T(r,t) = \sum_i C_i J_0\left(\frac{\beta_i}{R} r\right) \cdot e^{-\frac{a}{R^2}\beta_i^2 \cdot t}$$

- Regardless of the particular solution forms, in practically all solutions a **characteristic time**, called **thermal diffusion time**, can be identified:

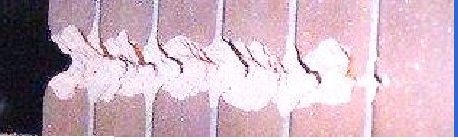
$$t_d = \frac{B^2}{a}$$

- The diffusion time is related to the **time required to reach a uniform temperature** in a region whose **relevant dimension** is **B** (e.g. the radius of a disk).

Diffusion time vs. Impact duration

- On the typical dimensions of interest (*several mm or more*), the **thermal diffusion time lasts from several to many milliseconds**.
- This is **much more than the duration of beam impacts** we are concerned about (**~ μs**). So the **assumption of instantaneous heat deposition looks** in general **appropriate**.
- However, it is important to note that **diffusion time can play a (beneficial) role in flattening down local temperature peaks on the sub-millimetric scale!!**

	Properties at Room Temperature				Diffusion Time [ms]		
Material	**Density [kg/m^3]**	**Specific Heat [J/kg/K]**	**Thermal Conductivity [W/m/K]**	**Diffusivity [mm^2/s]**	**L = 0.1 mm**	**L = 1 mm**	**L = 1 cm**
Copper (Glidcop)	8900	391	365	104.9	0.10	9.5	953
Tungsten Alloy (Inermet180)	18000	150	90.5	33.5	0.30	29.8	2983
Molybdenum	10220	251	138	53.8	0.19	18.6	1859
Titanium Alloy (Ti6Al4V)	4420	560	7.2	2.9	**3.44**	343.8	**34378**
Aluminum Alloy	2700	896	170	70.3	0.14	14.2	1423
Molybdenum-Graphite	2500	740	770	416.2	**0.02**	2.40	**240**
Graphite	1850	780.0	70	48.5	0.21	20.6	2061

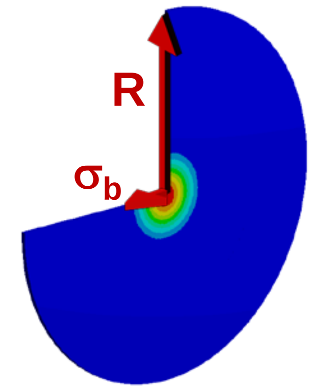


Diffusion time vs. Impact duration

- Transversal energy deposition profiles can often be approximated with a **Normal Gaussian Distribution**. Hence, initial temperature field in a disk or circular cylinder takes the form:

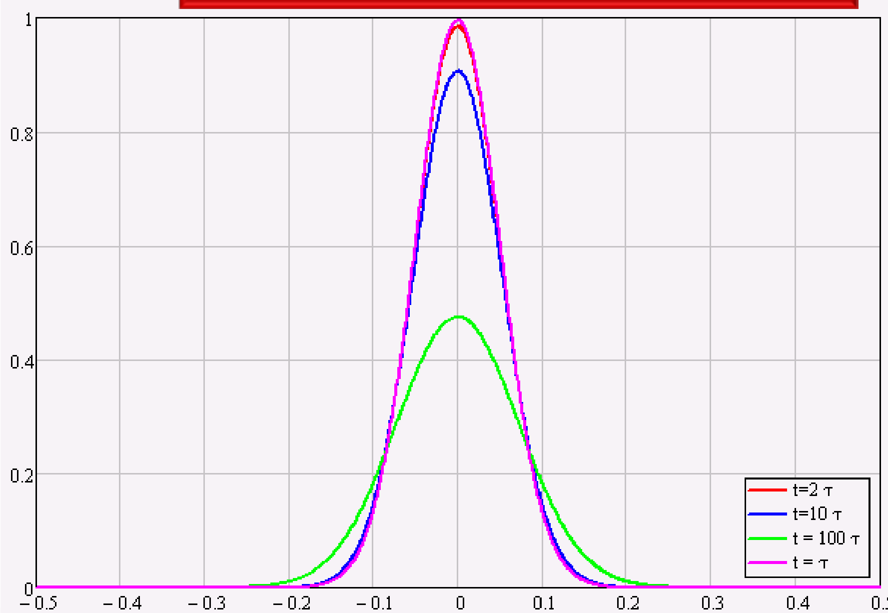
$$T(r,\tau) = T_0(r) = T_{\max} e^{-\frac{r^2}{2\sigma_b{}^2}}$$

where $\boldsymbol{\sigma_b}$ *is the distribution* ***standard deviation***

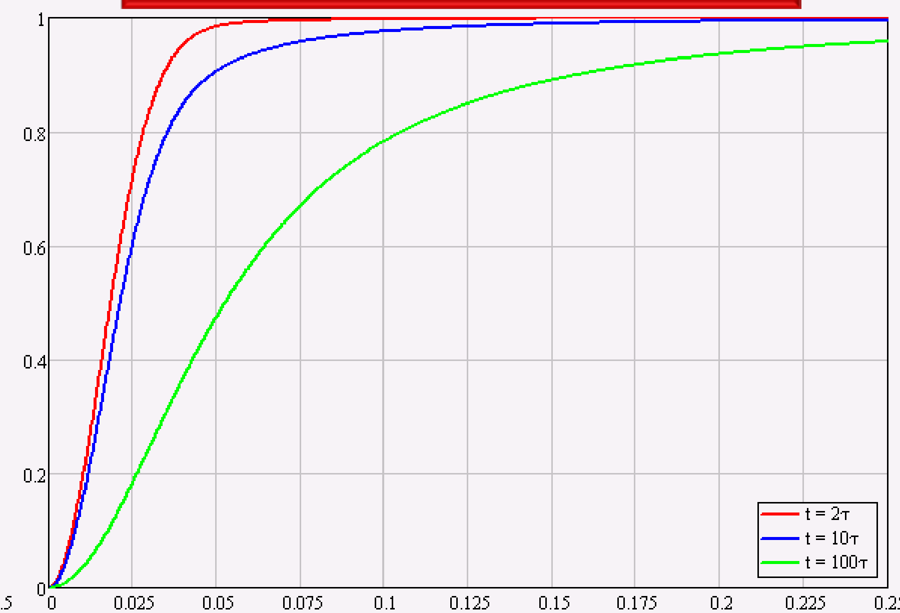


- **Example**: Temperature distribution in a graphite thin disk with radius *R = 5 mm*

Temperature distribution at various instants for σ_b = 0.25 mm

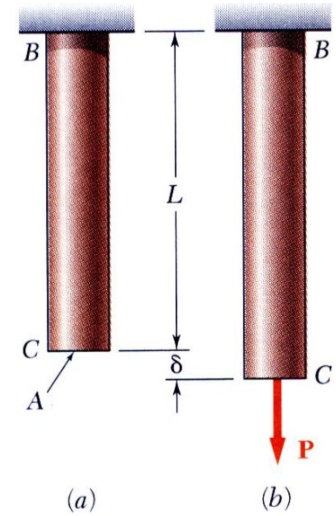


Temperature at disk center as a function of σ_b/R



- ✓ Linear Thermomechanical Analysis
  - ✓ Basics of Linear Thermoelasticty
  - ✓ Hooke’s law
  - ✓ Duhamel-Neumann equation
  - ✓ Quasistatic Thermal Stresses
  - ✓ Linear Dynamic Stresses

Linear Elasticity: Hooke's Law



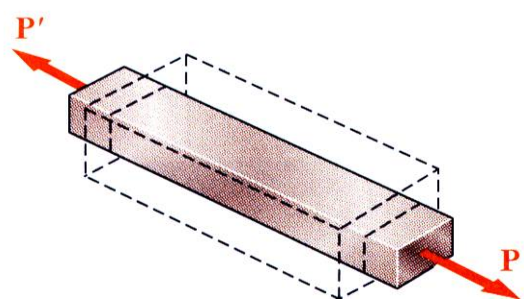
- Any body submitted to a mechanical **stress** (*force per unit area*) responds by deforming. The **ratio of** stress-induced **deformation to the initial dimension** is called mechanical **strain** $\boldsymbol{\varepsilon^M = \delta / L}$.
- In **linear elasticity** it is postulated that a **linear relationships** exists **between stresses and strains**.
  Mathematically this is expressed by the **Hooke's Law**. For an **isotropic** body, this reads:

$$\varepsilon_{ij}^{M} = \frac{1}{E}\left[(1+\nu)\sigma_{ij} - \nu\delta_{ij}\sigma_{kk}\right] \qquad (3)$$

where: ***E*** *is the* ***Young's Modulus*** *[Pa]*
ν *is the* ***Poisson's ratio*** *[-]*

- If only one component of normal stress is acting, this boils down to the well-know linear relationship:

$$\varepsilon_{11}^{M} = \frac{\sigma_{11}}{E} \qquad (3a)$$



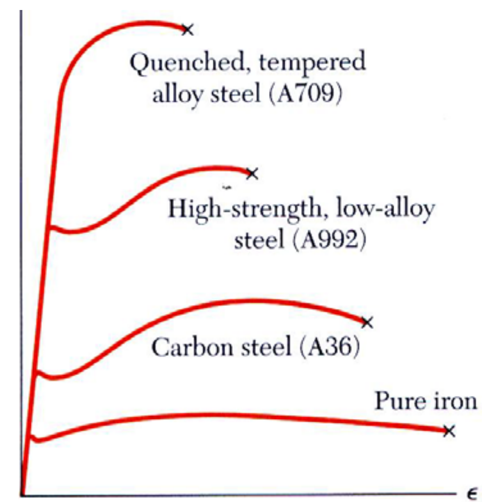
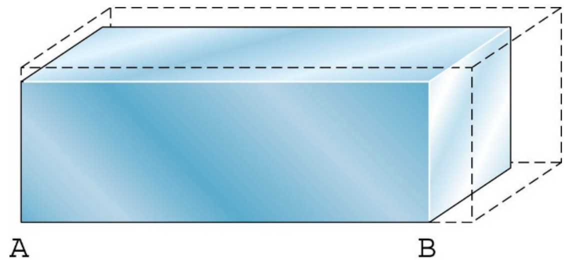
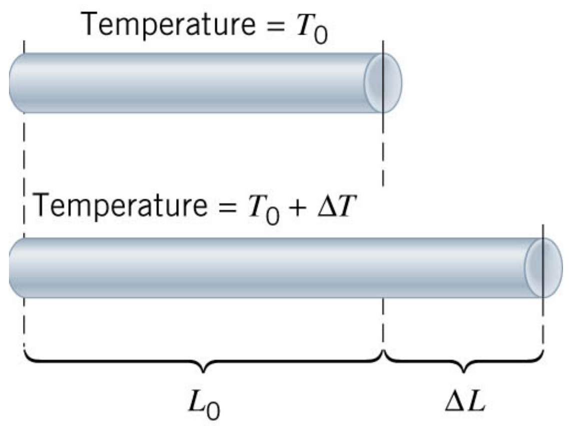


Fig. 2.16 Stress-strain diagrams for iron and different grades of steel.

- An **unrestrained body** submitted to a **change of temperature** undergoes a **thermal deformation**.
- If the body is **homogeneous** and **isotropic** and temperature change is **uniform**, this deformation is only volumetric (**shape is maintained**).
- Strains caused by thermal deformation on unrestrained bodies from an initial **reference temperature** (usually uniform and equal to ambient temperature), are called **free thermal strains** ε^T.
- The rate of linear change of dimension per unit temperature variation is called **Linear Coefficient of Thermal Expansion** (**CTE**) α



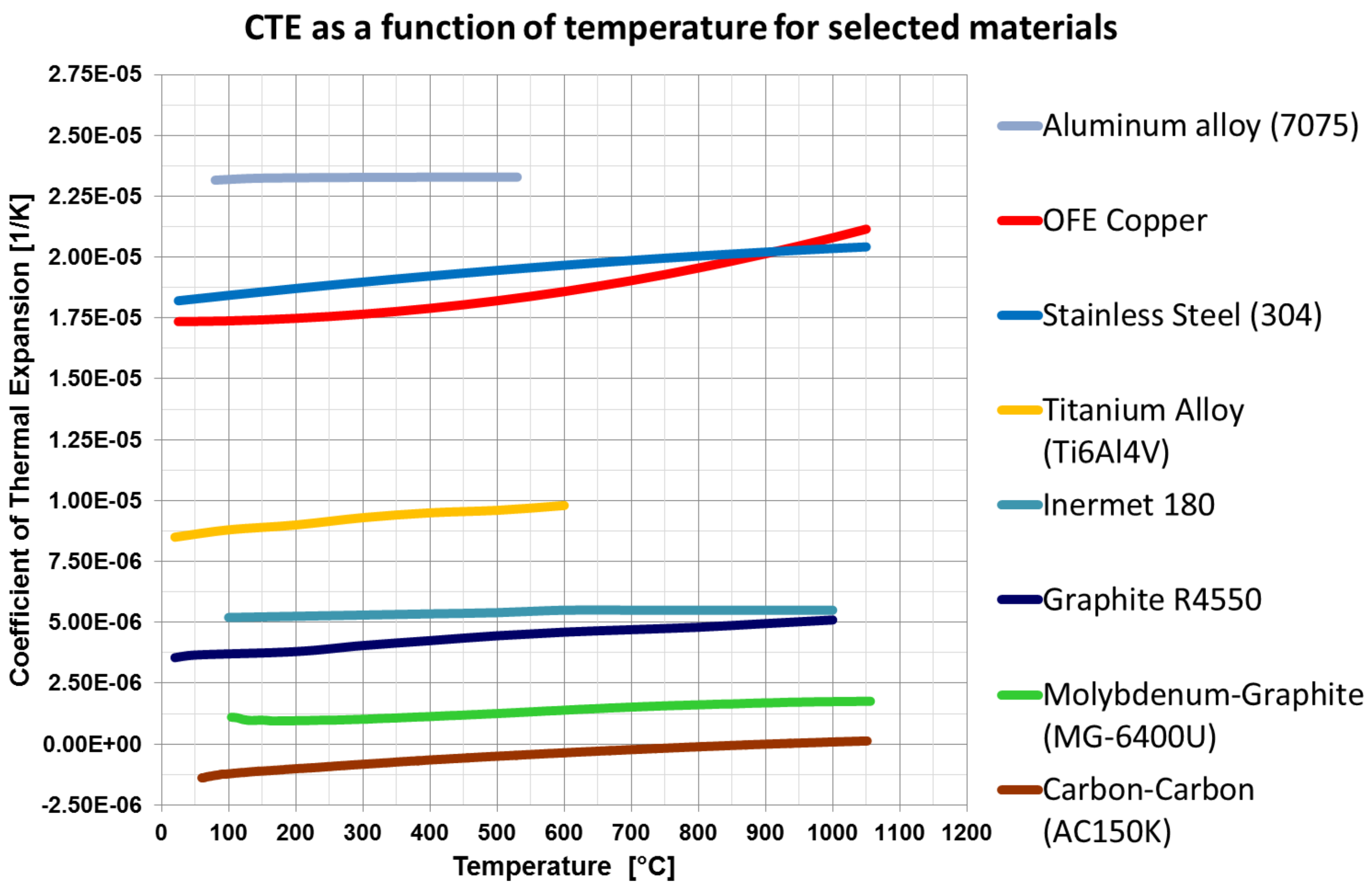


$$\alpha(\mathrm{T}) = \frac{dL}{LdT} \qquad [\mathrm{K}^{-1}]$$

- The Linear CTE is related to the **Volumetric Coefficient of Thermal Expansion** β ($\alpha = \frac{1}{3}\beta$)

Coefficient of Thermal Expansion

- **CTE** is in general **temperature-dependent**. This is particularly true at cryogenic temperatures (**below ~80 K, CTE tends to zero**)
- However, over limited temperature ranges above or about Room Temperature, it can be averaged to a constant value.



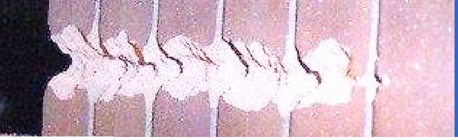
Linear Thermoelasticity: Duhamel-Neumann Law

- Hooke's law was extended by **Duhamel** and **Neumann** to include the first-order thermal effects (**Linear Thermoelasticity**)
- It assumes that **total strain** ε at a point consists of **mechanical strain** ε^M and **free thermal expansion** ε^T (**reference temperature** taken identically **equal to zero** for convenience).

$$\varepsilon_{ij} = \varepsilon_{\mathrm{ij}}^{\mathrm{M}} + \varepsilon_{\mathrm{ij}}^{\mathrm{T}} \quad \text{where} \quad \varepsilon_{\mathrm{ij}}^{\mathrm{T}} = \alpha\delta_{ij}T \quad \Rightarrow$$

$$\varepsilon_{ij} = \frac{1}{E}\left[(1+\nu)\sigma_{ij} - \nu\delta_{ij}\sigma_{kk}\right] + \alpha\delta_{ij}T \qquad (4)$$

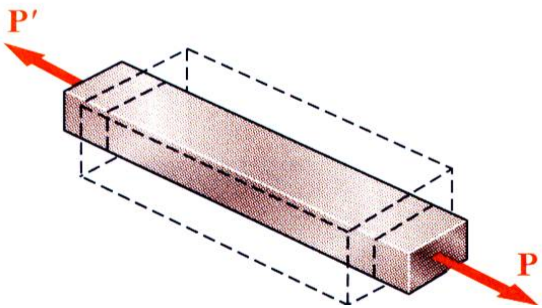
- In an isotropic body, **shear strains** are **never induced** by **free thermal expansion**.
- We can also observe that **the smaller the CTE**, **the smaller the thermal strains** and hence the total stresses.
- This is a **fundamental concept** in the design of **Beam Intercepting Devices**, since **thermal stresses are usually the most important load !!**
- ***No CTE, no Stress (regardless of the temperature increase … to some extent!!)***



- Inverting Eq. (4) **total stresses** can be obtained:

$$\sigma'_{ij} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-2\nu)\varepsilon_{ij} + \nu\delta_{ij}\varepsilon_{kk}\right] - \delta_{ij}\frac{E\alpha T}{1-2\nu} \qquad (5)$$

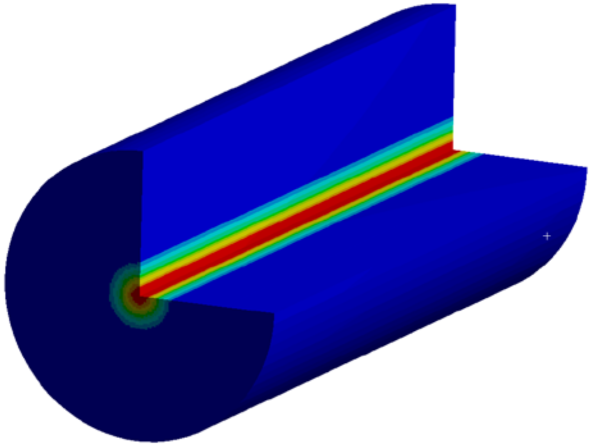
- **Stresses** may be induced by **mechanical loads**, spatial **nonuniformity in the temperature field** and/or **geometric restraints** preventing free thermal expansion.

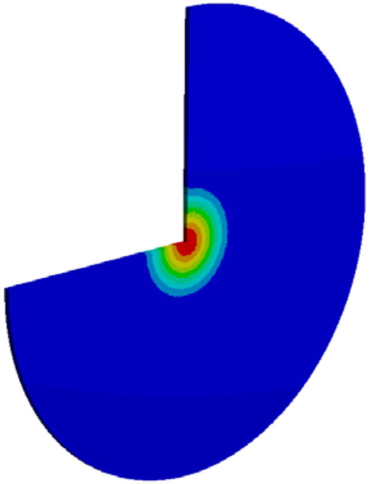


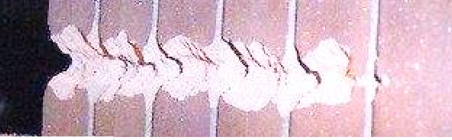
- Although the expression is **time-dependent** ($T = T(t)$), we obtain **quasistatic stresses** σ'_{ij} since mass inertia contributions are neglected!
- We get a succession of “snapshots” at various instants of the stress distribution, but **we can’t appreciate the dynamic effects (yet)**.

Quasistatic Thermal Stresses

- In **Beam Interacting Devices**, mechanical loads are usually negligible
- The design is (hopefully!) **isostatic**, free thermal expansion is hence allowed
- The **main** (often single) **source of stresses** is the **non-uniform temperature distribution** (and/or non-homogeneity of CTE, e.g. in case of composite structures …)
- **Quasistatic stresses** can be obtained combining **Eq. (5**) with the **equations of equilibrium**, **compatibility equations** and **boundary conditions**
- Analytical solutions are available for special cases.
- The **most useful** for beam-induced thermal shocks are those obtained for **long circular cylinders** and **thin disks** which can reasonably approximate **targets, dumps, absorbers, windows** etc.

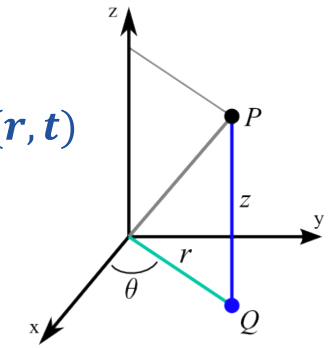






Quasistatic Stresses in Cylinders and Disks

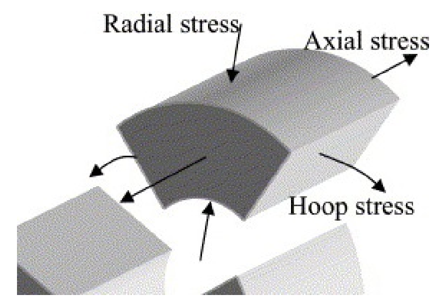
- For **cylinders** and **disks** a cylindrical reference system is used.
- In case of **axially symmetric, z-independent thermal distribution** $\boldsymbol{T(r,t)}$ with **adiabatic boundary conditions,** we get for the **radial** and **hoop (circumferential) stresses**:



$$\sigma_r'(r,t) = \frac{E\alpha}{\zeta}\left[\frac{1}{R^2}\int_0^R T(r,t)r\,dr - \frac{1}{r^2}\int_0^r T(r,t)r\,dr\right] \quad (6a)$$

$$\sigma_\theta'(r,t) = \frac{E\alpha}{\zeta}\left[\frac{1}{R^2}\int_0^R T(r,t)r\,dr + \frac{1}{r^2}\int_0^r T(r,t)r\,dr - T(r,t)\right] \quad (6b)$$

with $\zeta = 1$ for disks and $\zeta = 1 - \nu$ for cylinders



Notes:

*1. At $\boldsymbol{r = 0}$ **radial** and **circumferential stresses** are **identical** (compressive) (the second term tends to $\frac{1}{2}T_0(t)$)*

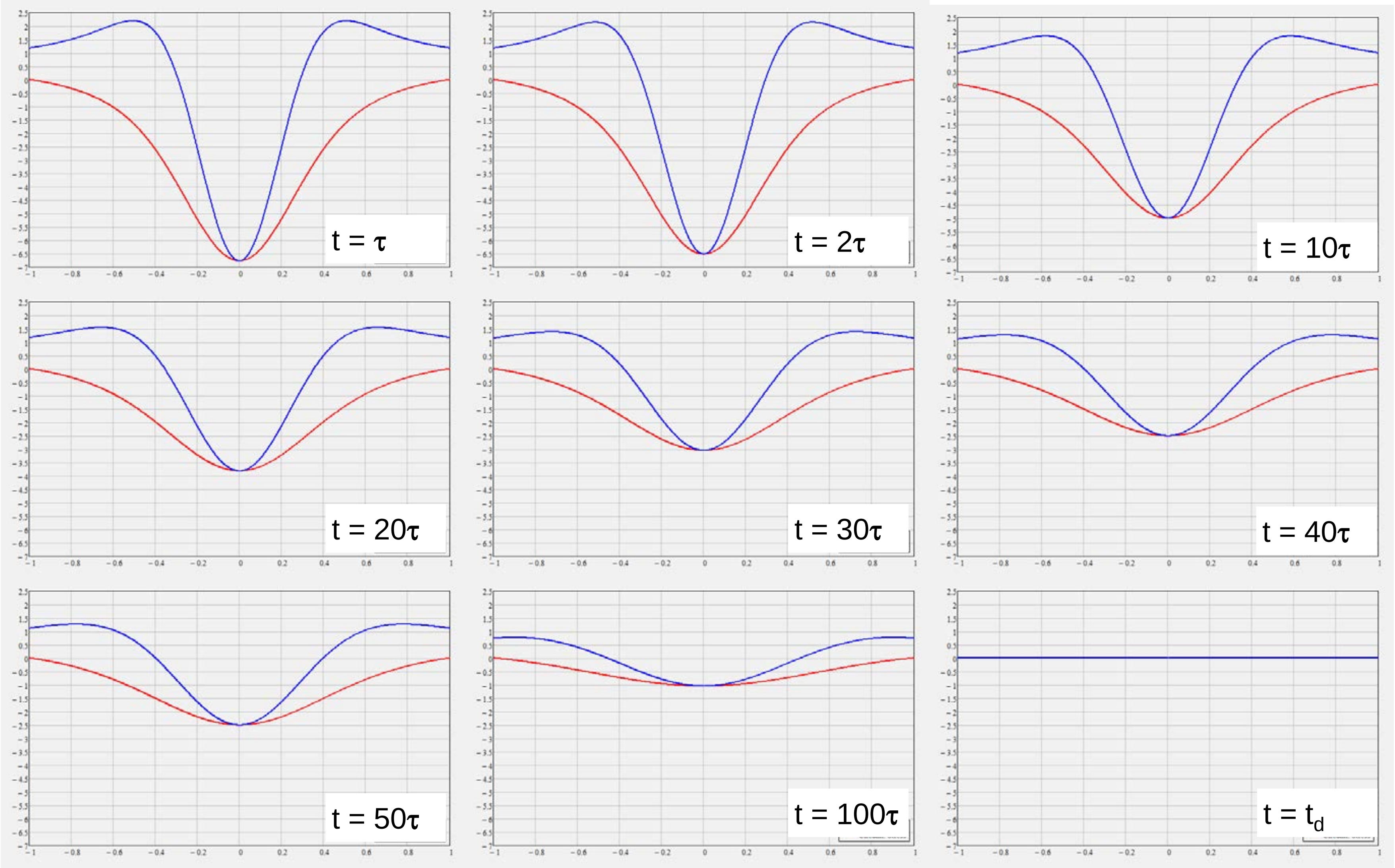
*2. At $\boldsymbol{r = R}$ **radial stress is zero**, while **hoop stresses** are always $\geq \mathbf{0}$.*

3. Since the first term is proportional to the Total Deposited Energy and remains constant for an adiabatic problem, stresses at center and outer rim can be easily computed regardless of the actual temperature distribution!

*4. At $\boldsymbol{t \gg t_d}$, when T becomes uniform, **radial and hoop stresses** go to **0 everywhere***

Quasistatic Stresses in Cylinders and Disks

Radial and **Circumferential stresses** for a Normal Distribution at various instants in time



Quasistatic Axial Stresses in Slender Bodies

- For **thin structures** such as disks, **through-thickness stresses are negligible**.
- However, for **long**, **slender structures** (*rods*, *bars*, *beams*) **axial stresses are usually very important**.
- To compute latter components, we initially assume that the structure is **restrained at its ends**, i.e. **axial strain is zero** throughout ($\boldsymbol{\varepsilon_z = 0}$)
- In this hypothesis, quasistatic axial stresses can easily be derived from Eq. (4). E.g., in cylindrical coordinates:

$$\sigma'_z = \nu\left(\sigma'_r + \sigma'_\theta\right) - E\alpha T \qquad (8)$$

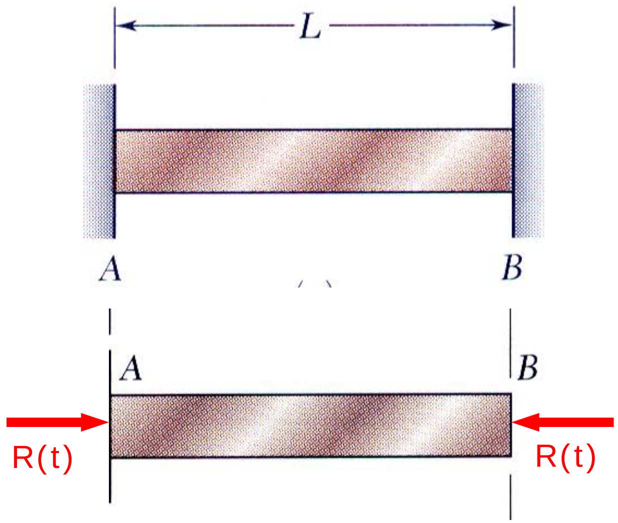
- This distribution of stresses results in a **compressive force $\boldsymbol{R(t)}$** applied at the ends, required to suppress thermal expansion $\boldsymbol{\delta_T}$, which for $t \geq \tau$ becomes:

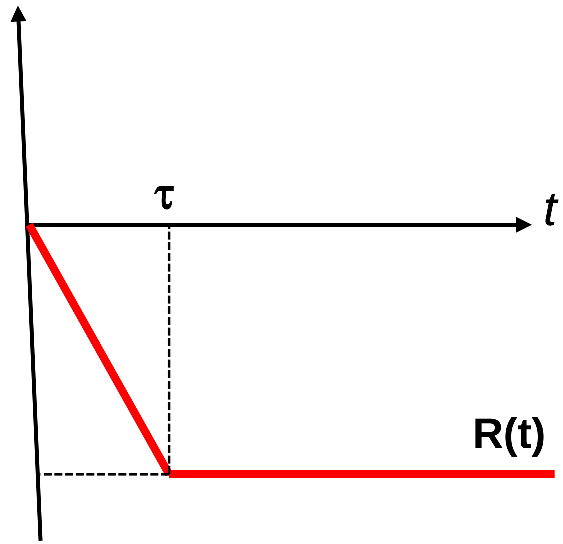
$$R(t) = 2\pi\int_0^R \sigma'_z(r,t)dr = -E\alpha\frac{Q_d}{\rho c_p} = -E\alpha T_F \pi R^2 \qquad (9)$$

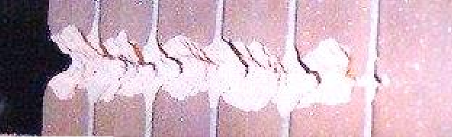
*where $\dot{\boldsymbol{Q}}_{\boldsymbol{d}}$ is the **Total Deposited Energy per unit length** [J m^{-1}]*

- **Homework**: making use of Eqs. (6a), (6b) and (8), prove Eq. (9)

- From Eq. (9) we observe that the resultant axial force $\boldsymbol{R}(\boldsymbol{t})$ is **proportional to the Total Deposited Energy** (per unit length) $\boldsymbol{Q_d}$
- Also, since $\boldsymbol{Q_d}$ is conserved after the impact (the problem is adiabatic), for $t \geq \tau$, $\boldsymbol{R}$ is constant and proportional to the **final uniform temperature $\boldsymbol{T_F}$, regardless of deposited energy distribution**
- For $t < \tau$, $\boldsymbol{R}(\boldsymbol{t})$ follows the trend of the deposited energy, so it can typically assumed to grow linearly from zero (*approximation of the actual bunched structure*).

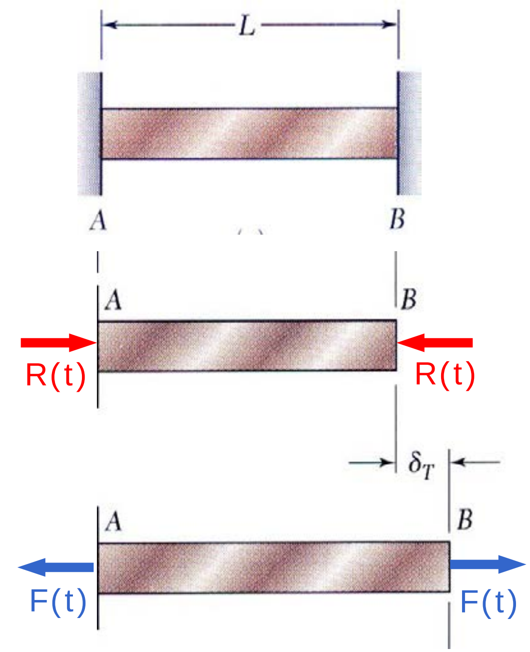


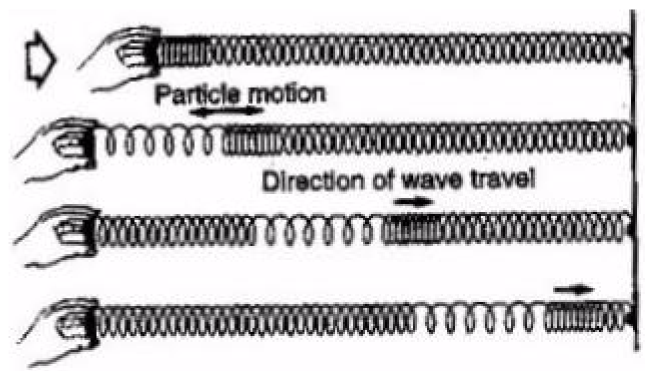




Introduction to Dynamic Stresses

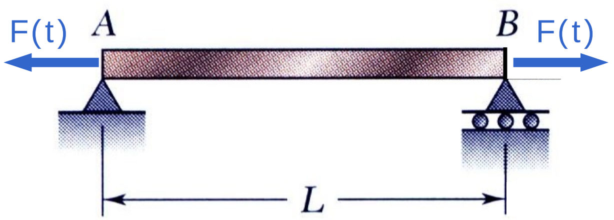
- If the structure is free to expand (as usually is), we can **superpose** a **traction force** $\boldsymbol{F(t) = -R(t)}$ to restore the **free end boundary condition** and allow thermal expansion $\delta_{\boldsymbol{T}}$.
- For <u>slender structures</u>, the **dynamic axial stresses** $\boldsymbol{\sigma''_z}$ can be reduced to the **mechanical response** to a **pulse excitation** $\boldsymbol{F(t)}$ with **rise time** τ.
- Since τ is very short, **thermal expansion is prevented** in the bulk material **by mass inertia**.
- Expansion begins **from the two rod ends** generating **elastic stress waves** travelling at **the speed of sound** $c_0 = \sqrt{E/\rho}$ towards the center.
- The complete response of the system can be obtained by <u>**superposing dynamic stresses $\boldsymbol{\sigma''}$ to quasistatic stresses $\boldsymbol{\sigma'}$**</u> obtained for a restrained structure!





$$\sigma_z = \sigma'_z + \sigma''_z \qquad (10)$$

- The mechanical response of a cylindrical rod to a pulse with rise time is a well-know problem of **theory of vibrations**.



- It can be solved resorting e.g. to the **mode-summation method.** The axial displacement $\boldsymbol{u(z,t)}$ is **expanded** in terms of longitudinal natural modes $\boldsymbol{\phi_z(z)}$ and generalized coordinates $\boldsymbol{q_z(t)}$.

$$u(z,t) = \sum_i \phi_{z_i}(z) q_{z_i}(t) \qquad (11)$$

- Solution can be obtained by means of **Lagrange's Equation** for each independent mode:

$$\frac{d^2 q_{z_i}}{dt^2} + \omega_i^2 q_{z_i} = Q_i$$

*where **Generalized Forces, Natural Frequencies** and **Natural modes** are:*

$$Q_{z_i}(t) = \frac{F(t)\sqrt{2}}{m}\left[1-(-1)^i\right]$$

$$\omega_{z_i} = \frac{i\pi}{L}\sqrt{\frac{E}{\rho}} \qquad \phi_{z_i}(z) = \sqrt{2}\cos\left(i\frac{\pi z}{L}\right)$$

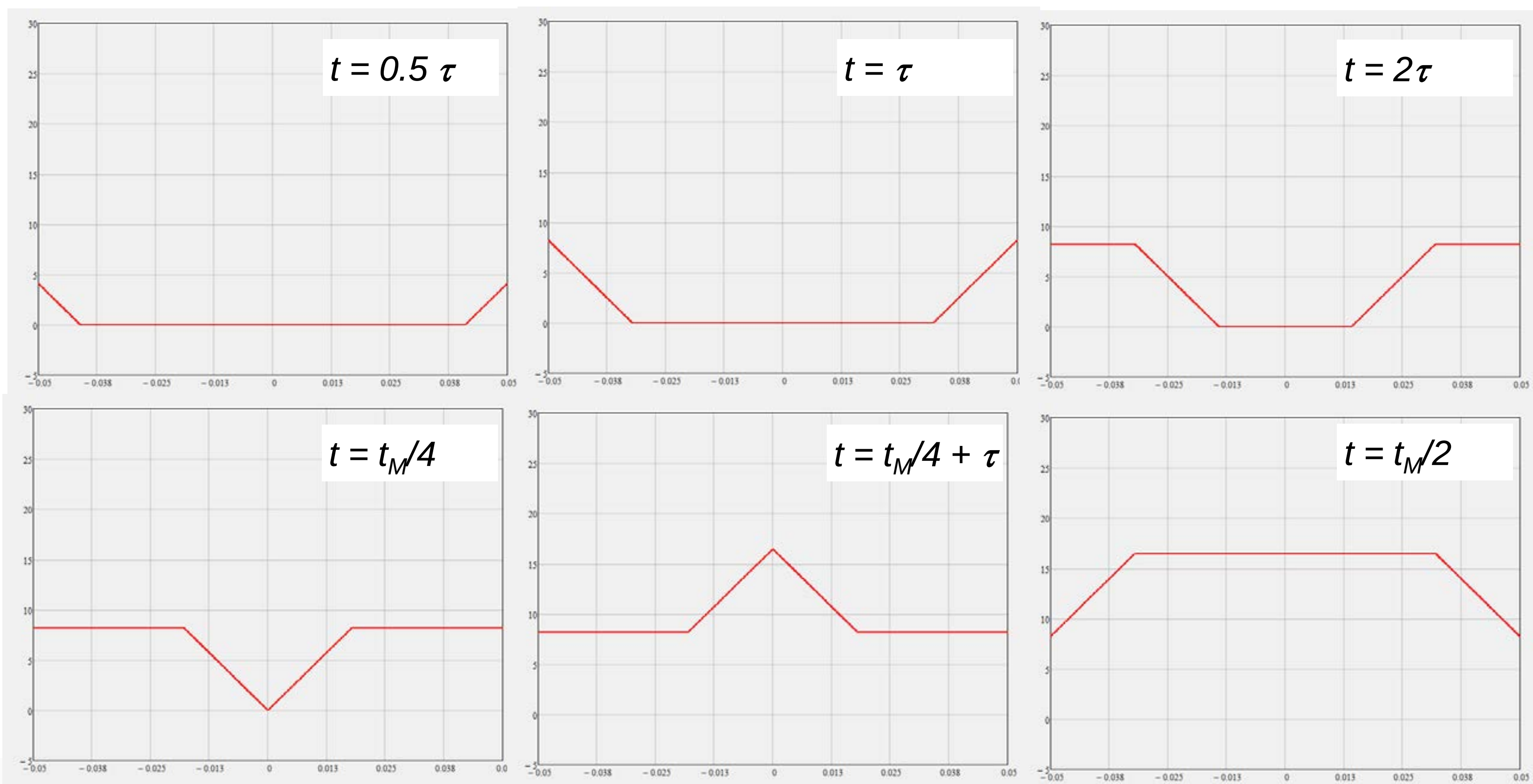
- We finally get:

$$q_{z_i}(t \geq \tau) = \frac{F_{z_i}}{(\omega_{z_i})^2} \cdot \left(1 - \frac{\sin(\omega_{z_i} t)}{\omega_{z_i}\tau} + \frac{\sin[\omega_{z_i}(t-\tau)]}{\omega_{z_i}\tau}\right)$$

$$q_{z_i}(t < \tau) = \frac{F_{z_i}}{(\omega_{z_i})^2} \cdot \left(\frac{t}{\tau} - \frac{\sin(\omega_{z_i} t)}{\omega_{z_i}\tau}\right)$$

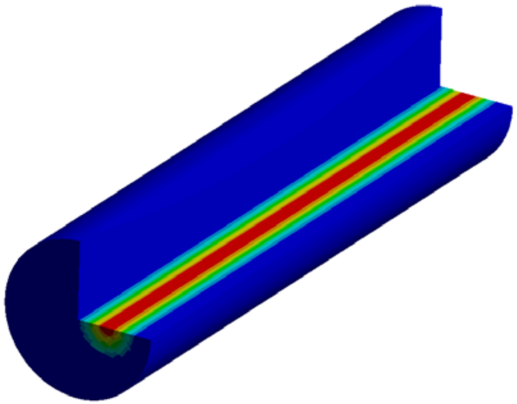
Dynamic Axial Stresses

- Axial strain can be obtained from displacement by simple derivation. Axial stress is simply obtained by multiplying by the Young's modulus.
- At the beginning, a tensile stress wave starts travelling at the speed of sound c_0 from both ends while force $F(t)$ linearly builds up. At $t = \tau$, $F(t)$ stops growing and so does stress. The axial stress wave reaches the rod center after one quarter of the **wave period** given by $t_M = \frac{2L}{c}$

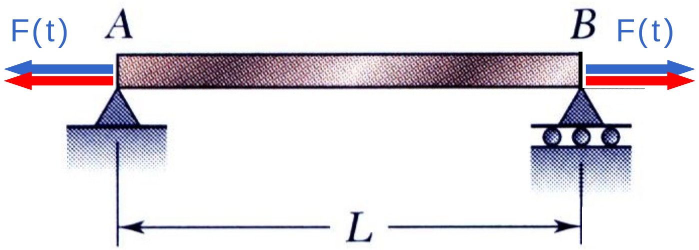


- A similar approach can be used for **dynamic radial stresses** (although the treatment is more cumbersome).
  However for slender structures these are usually negligible!

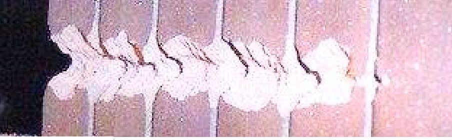
- **Question**: What would happen to a long rod free to expand if it were impacted by the same energy distribution offset from its axis?



- **Hint**: remind the case of bent Beryllium rod …



- ✓ Non-linear Thermomechanical Analysis
  - ✓ Thermally Induced Dynamic Regimes
  - ✓ Introduction to Non-linear Numerical Codes
  - ✓ Dynamic Plastic Regime
  - ✓ Shockwave Regime
  - ✓ Constitutive Models
  - ✓ Hydrodynamic Tunnelling
  - ✓ Thermal Shocks vs. Mechanical Impacts



Thermally-induced Dynamic Regimes

- High-energy accelerator components are usually designed to work in the **Elastic Regime** we have analyzed so far …

HOWEVER

- Beam impact accidents can provoke **permanent deformation of the component**…⇨

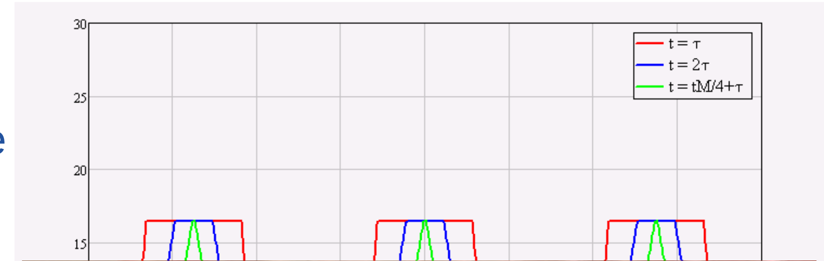
Plastic Regime

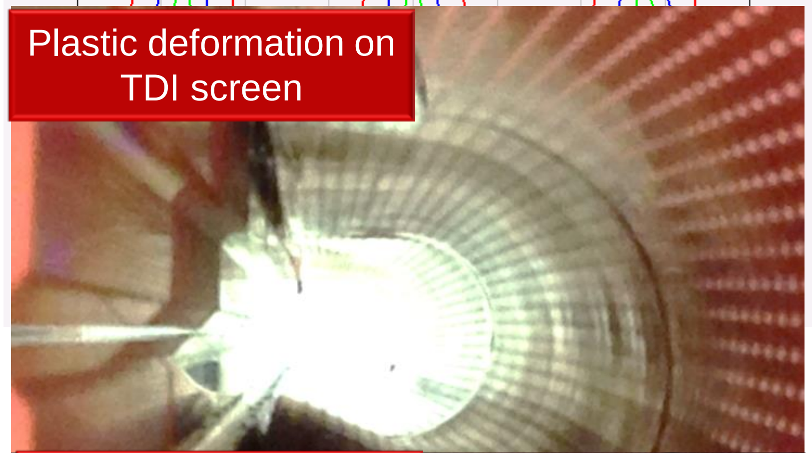
  - *Small (negligible) changes in material density*
  - *Irreversible Plastic deformations*
  - *Stress waves slower than speed of sound*

- …or even its **catastrophic failure**, if deposited power is high enough … ⇨

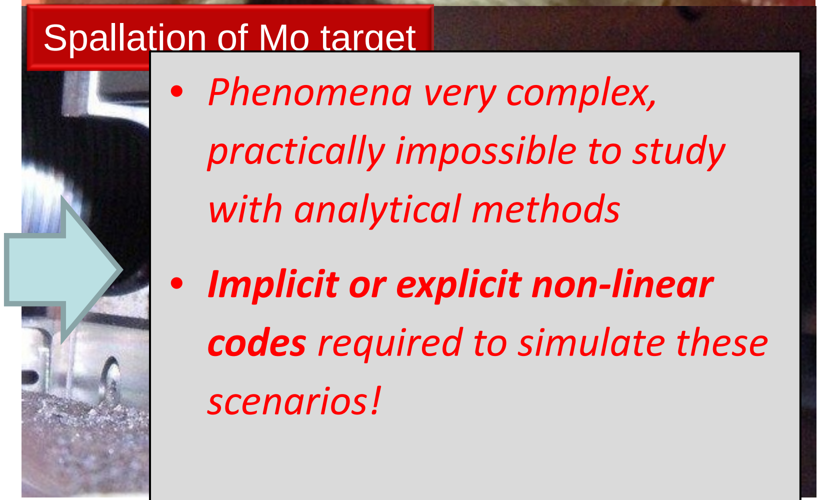
Shockwave Regime

  - *Intense stress waves faster than speed of sound*
  - *Large changes of density*
  - *Phase transitions*
  - *Explosions, material projections, spallation …!*





Plastic deformation on TDI screen



Spallation of Mo target

- *Phenomena very complex, practically impossible to study with analytical methods*
- ***Implicit or explicit non-linear codes** required to simulate these scenarios!*

Numerical Codes: Time Integration Methods

Implicit time integration schemes:

- **Unconditionally stable**
  bigger time steps are allowed, although limited to accurately capture dynamic response.
- **Numerical damping** can occur.
- **Computationally expensive**
  requires matrix inversion.

Better for stable, long duration or quasi-static phenomenon (e.g. dynamic response and final deformation).

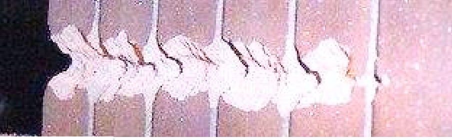
Standard F.E.M. codes
(Ansys, Abaqus, Nastran, …)

Explicit time integration schemes:

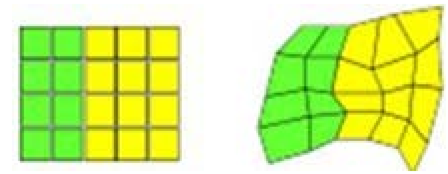
- **Conditionally stable**
  the time step must be chosen according to the element dimension (CFL condition) for the scheme stability.
- **Computationally efficient**
  no matrix inversion required, only multiplication.

Better for capturing large variations occurring in a very short time (e.g. rapid change of phase during beam impact).

Hydrocodes
(Autodyn, LS-Dyna, BIG2 …)



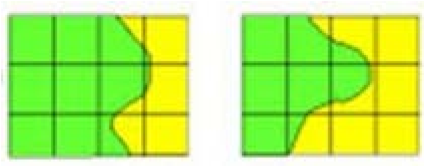
Numerical Codes: Mesh Schemes



Lagrangian mesh: it moves and distorts with the material it models as a result of for

Standard F.E.M. codes and Hydrocodes

- The most efficient so
- Very slow when an element incurs in large deformations.

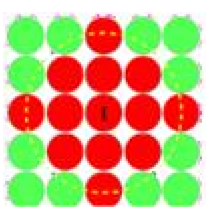


Eulerian mesh: it consists of a fixed mesh, allowing materials to flow from one element to the next.

- Very … treme material move…

Hydrocodes and CFD

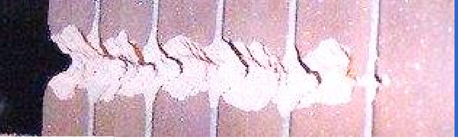
- Computationally intensive and requiring higher element resolution.



SPH (smooth particle hydrodynamic) mesh: it is a mesh-free method ide… ms with extensive material da

Hydrocodes

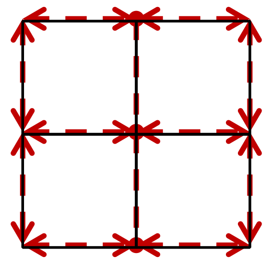
- Possibility to study the crack propagation inside a body and the motion of expelled fragments/liquid droplets.

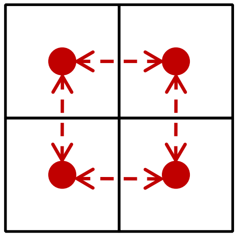


Numerical Codes: Hybrid mesh (Hydrocodes)

- **Hydrocodes** are **highly nonlinear wave propagation tools**, initially developed for high speed mechanical impacts where solids could be approximated by a **fluid-like behavior**.
- Simulations can be performed using two different meshing methods:

Lagrangian mesh: interconnected **multi-nodal elements** with shared external nodes, used for far-from-impact regions.

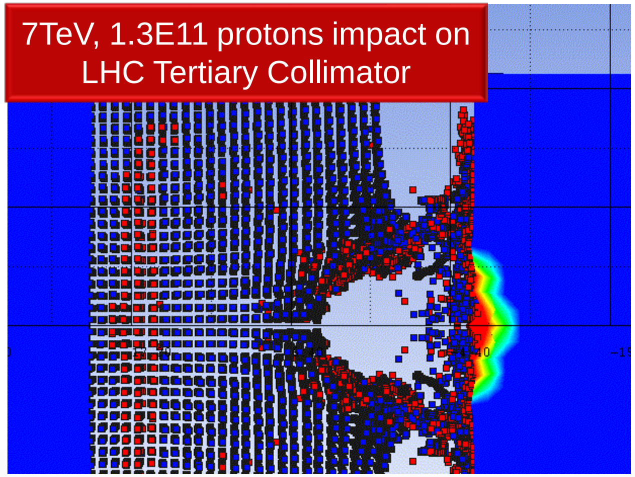




SPH mesh: **single node** elements interacting with each other, used for near-to-impact regions.

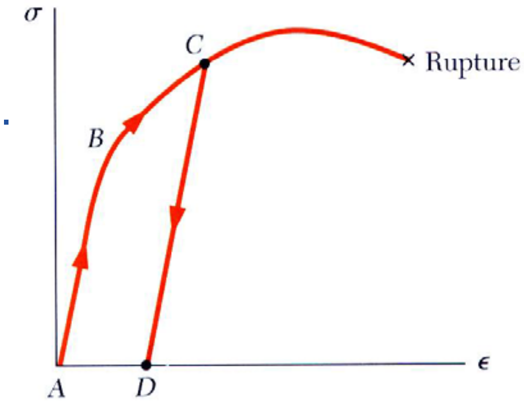
SPH (**Smoothed Particle Hydrodynamics**) elements dimensions:
The SPH elements must be generally very small to accurately model the material. Compromise to be found between accuracy and computation time.

Interaction SPH - Lagrangian mesh:
When an SPH particle approaches a Lagrangian part the interaction matrix must take into account the non penetration of solids and turn kinetic energy into deformation.

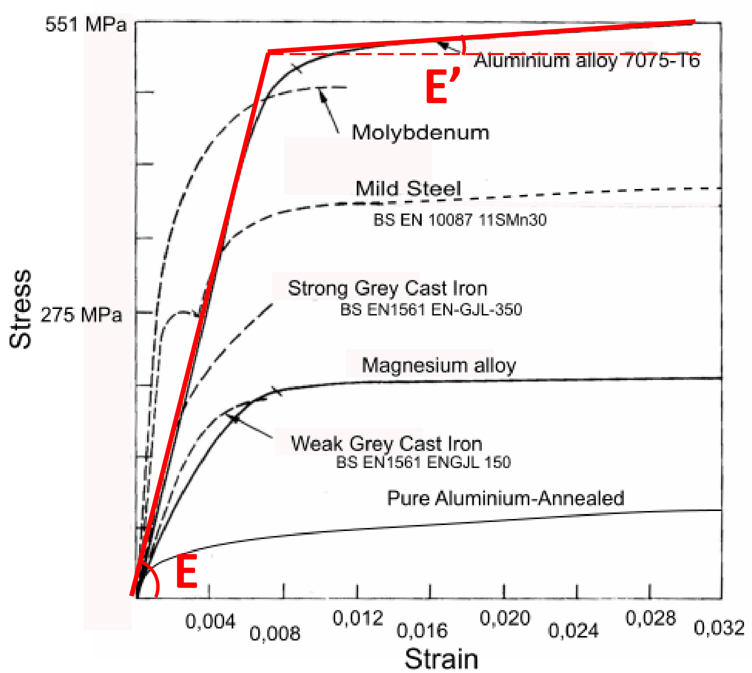


Dynamic Plastic Regime

- A material is **plastically** deformed when it undergoes **permanent changes of shape** in response to applied forces ..
- Stress-strain curve usually becomes strongly non-linear …



- *…however, in some cases the problem can be simplified, approximating material behavior with a bilinear hardening law:*

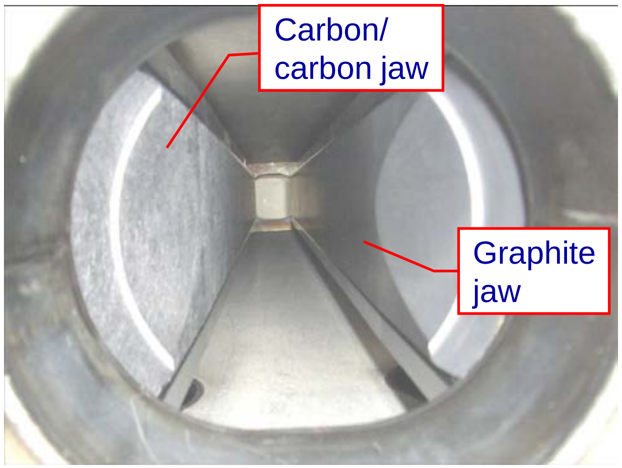


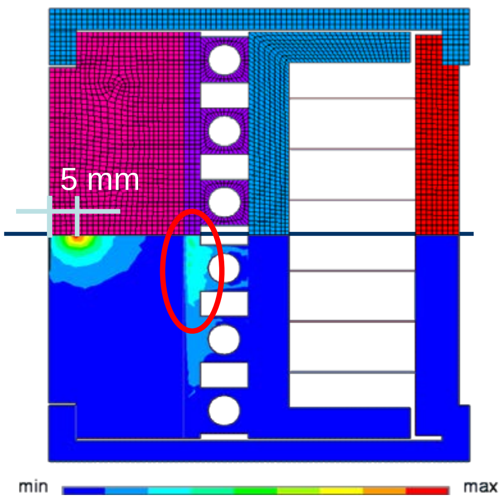
$$\sigma = E\varepsilon_{el} + E'\varepsilon_{pl} \qquad (9)$$

- ***E*** *is the* ***Young's Modulus***
- ***E'*** *is the slope of the plastic linear function, sometimes called* ***Tangent Modulus***
- *If* $E' = 0$ *the material is* ***elastic-perfectly plastic****.*

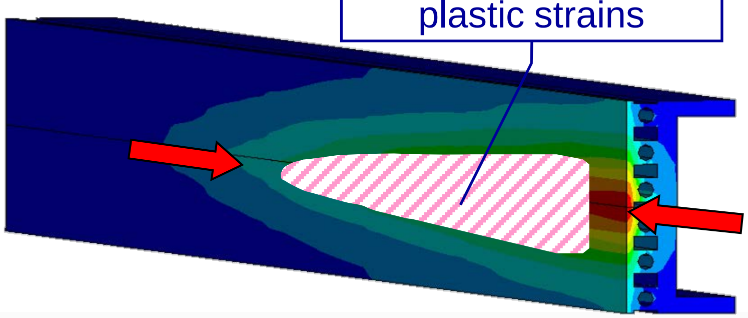
- In plastic regime, an implicit FEA code (e.g. ANSYS) is usually adopted to simulate structure response.

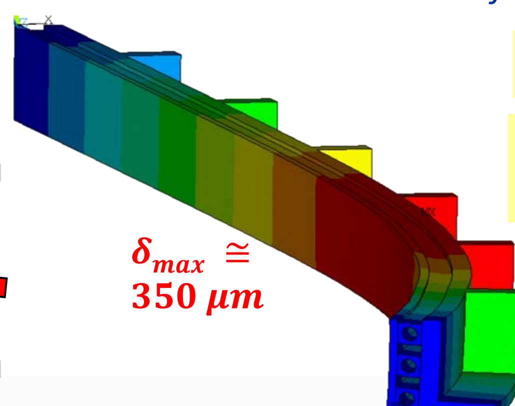
Example: LHC Secondary Collimator submitted to robustness test in 2004 (288 x 1.15x10^{11} p bunches, 450 GeV)





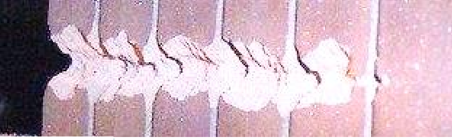
- *3D coupled analysis to assess* ***temperature, stresses and strains***
- *Priority given to critical carbon-based jaw blocks ⇨* ***post-mortem analysis confirmed survival of both blocks.***
- *A moderate* ***T increase*** *(~70°C) on OFE-Cu back-plate was* ***initially ignored …***
- *A simple analytical check anticipated what numerical simulation then confirmed…*





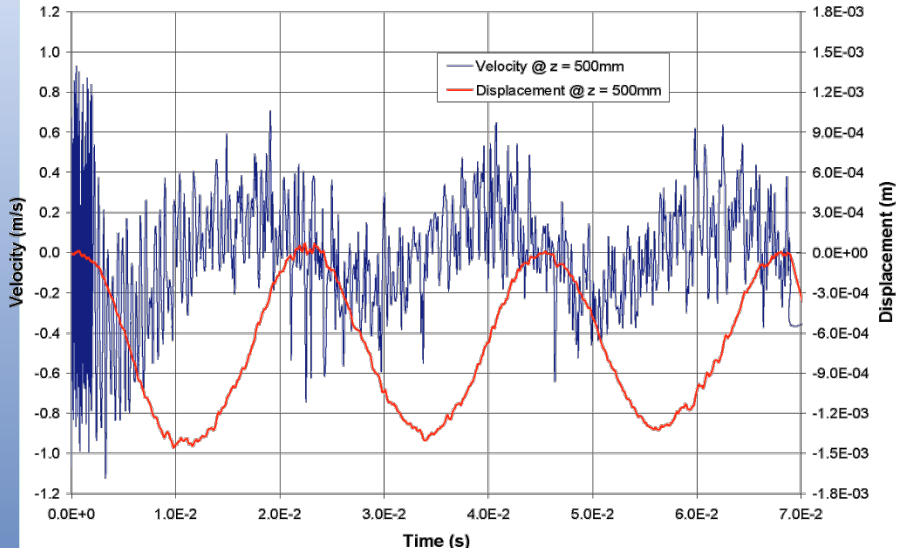
$$\varepsilon_{z_{\max}} = -\alpha \Delta T_{\max} \cong -0.0012$$

$$\sigma_{z_{\max}}^{lin} = -\frac{E \alpha \Delta T_{\max}}{1-\nu} \cong -210 MPa$$

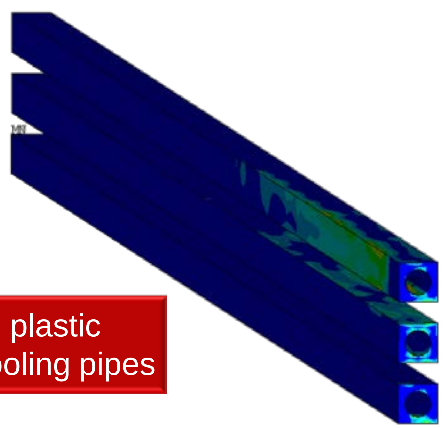


Dynamic Plastic Regime: Example

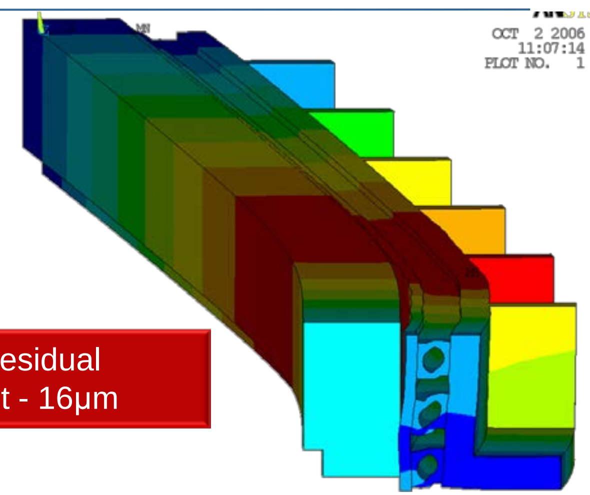
Example: 3D Thermo-Mechanical Elastic-Plastic Analysis of same collimator after design upgrade (from **OFE-Copper** to **Glidcop**)



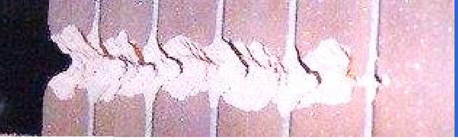
- *1st frequency of **flexural oscillation** ~45Hz with a max. **amplitude** of **1.5mm***
- *Since stresses acting on the structure slightly exceed elastic limit only on a small region, the residual plastic deformation should be limited*



A small amount of residual plastic deformation is found on cooling pipes

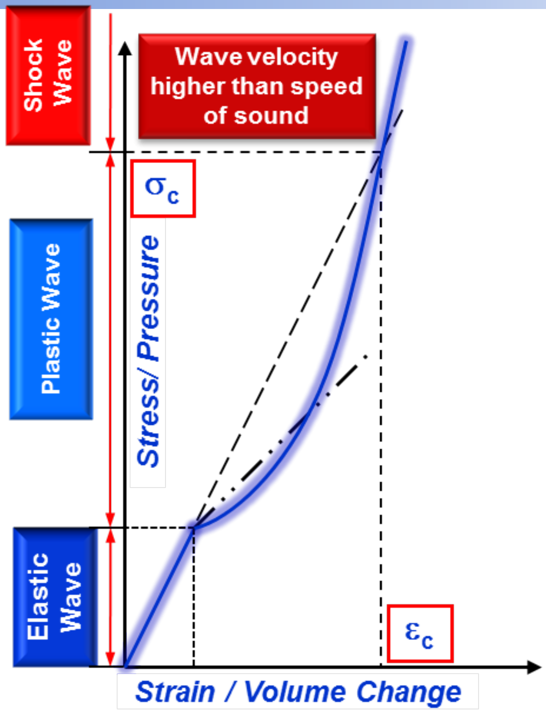


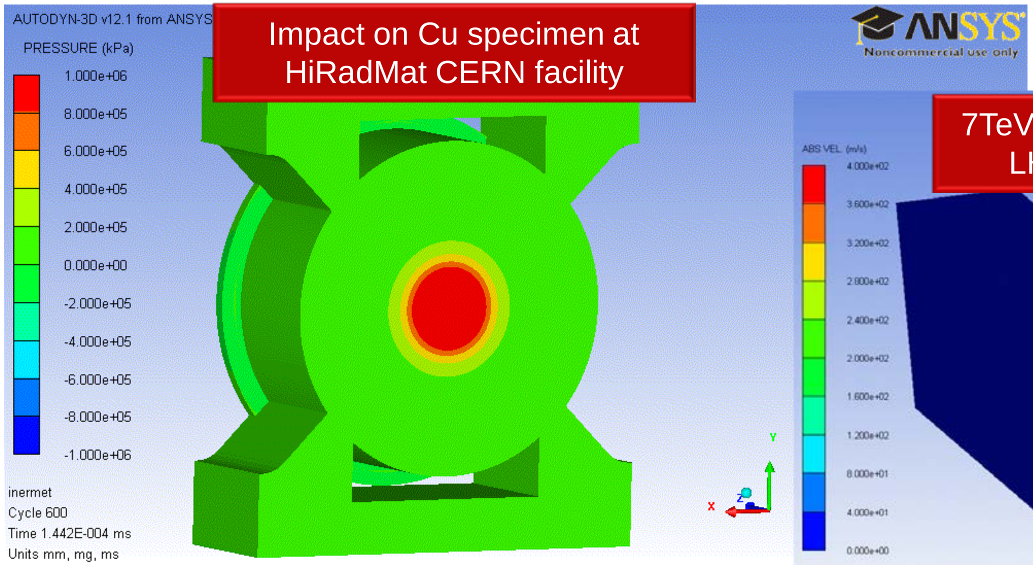
Transverse residual displacement - 16µm

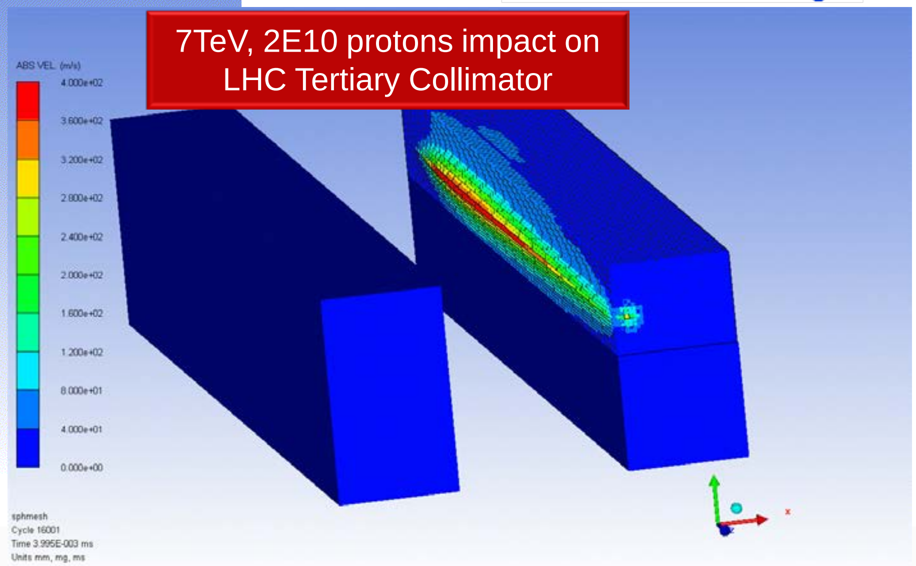


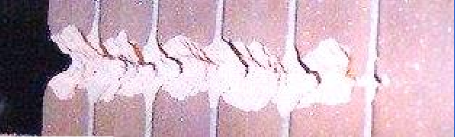
Shockwave Regime

- When the impact induces **large density and phase changes**, classical **Structure Dynamics** approach (analytical or numerical) is **no longer viable**.
- In these regimes, materials tend to behave like fluids ⇨ Hydrodynamic approach ⇨ **Hydrocodes**
- Complex material **Constitutive Models** are required, i.e. **Equations of State, Strength Model** and **Failure Model**.



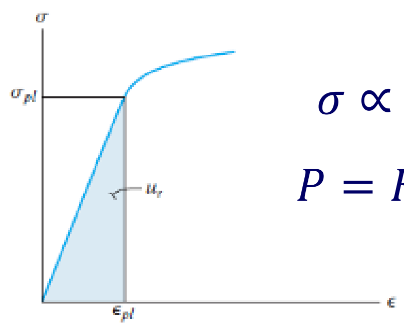






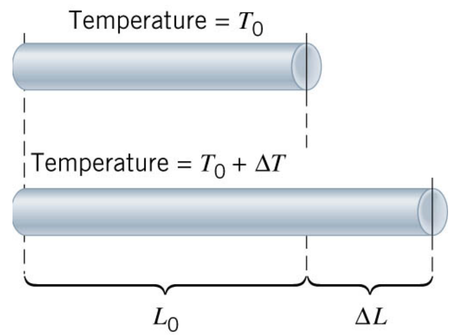
Constitutive Models: Equation of State

- Relations of Linear Thermoelasticity between **pressure** (stress), **density** (specific volume) and **temperature** (internal energy) are replaced by the **Equation of State (EOS)**.



$$\sigma \propto E \cdot \varepsilon$$

$$P = K \cdot \frac{\Delta v}{v}$$



$$\varepsilon \propto \alpha \cdot \Delta T$$

$$\frac{\Delta v}{v} = \beta \cdot \Delta T$$

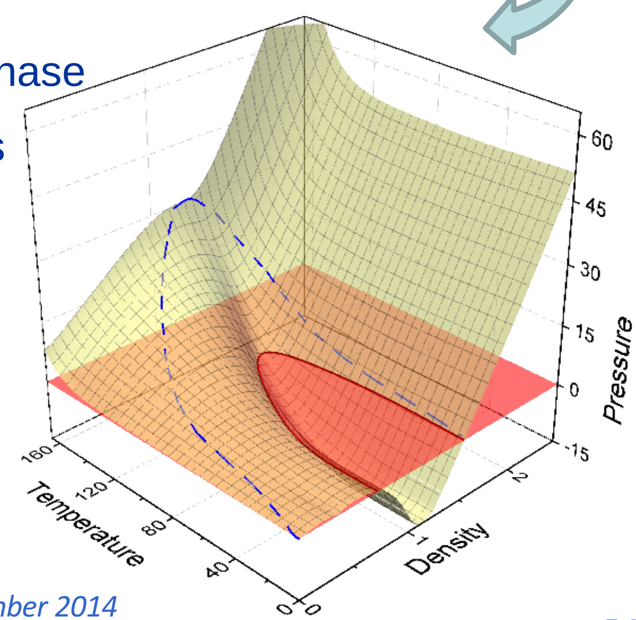
- EOS correlates pressure, density and temperature (or energy) for a given material over a wide range of values.
  - **Analytical**, e.g. Mie-Grüneisen defined for one single phase
  - **Tabular**, e.g. SESAME encompassing phase transitions

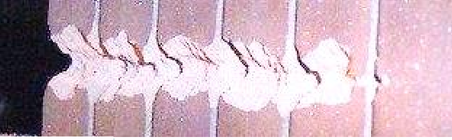
$$P_c = A_1\mu_c + A_2\mu_c^2 + A_3\mu_c^3 + (B_0 + B_1\mu_c)\rho_0 e$$

$$P_T = T_1\mu_T + T_2\mu_T^2 + B_0\rho_0 e$$

$$\mu = \frac{\rho - \rho_0}{\rho_0} \qquad E = \rho_0 e$$

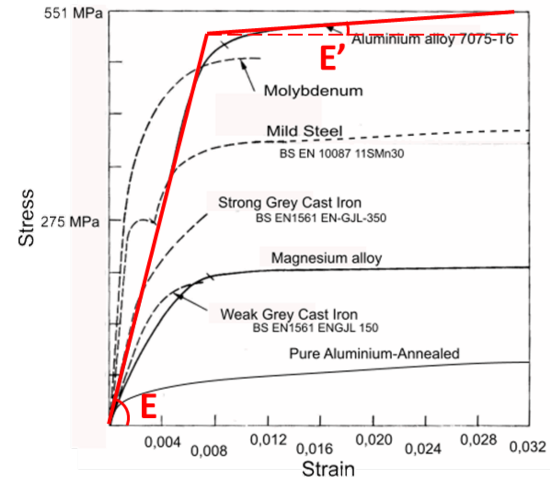
Example:
Generalized Mie-Grüneisen
(only valid for solid phase in tension and compression)





Constitutive Models: Strength Model

- Bilinear plasticity models have a certain number of limitations:
  - Usually derived at **Room Temperature**
  - Plastic-curve **changes of slope are disregarded**
  - **Strain-rate hardening is neglected** → no difference in the model between static and dynamic load application!



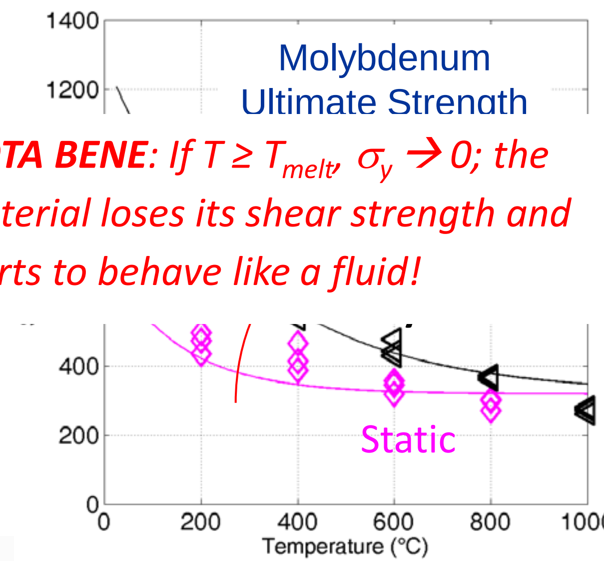
- Multiparameter **strength models** were developed, trying to take into account effects of *plastic flow*, *strain rate* and *temperature*

- **Johnson-Cook** model is particularly suitable for metals and ductile materials.

Johnson-Cook

$$\sigma_y = \left(A + B\varepsilon_{\text{pl}}^n\right)\left(1 + C\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)\left[1 - \left(\frac{T - T_{\text{room}}}{T_{\text{melt}} - T_{\text{room}}}\right)^m\right]$$

Strain Hardening | **Strain-Rate Hardening** | **Thermal Hardening/Softening**



***NOTA BENE**: If $T \geq T_{melt}$, $\sigma_y \rightarrow 0$; the material loses its shear strength and starts to behave like a fluid!*

Constitutive Models: Failure Model

- Plastic flow is computed by the strength model up to **material failure**
- Single-parameter material strength used in standard codes is replaced by complex **Failure Models** based on **damage accumulation theories**
- **Different models for different failure mechanisms and materials!**

Ductile Failures on high deformable materials (e.g. Cu)

Johnson-Cook Failure

$$D = \sum \frac{\Delta \varepsilon}{\varepsilon^f}$$

$$\varepsilon^f = \left[D_1 + D_2 e^{D_3 \sigma^*}\right]\left[1 + D_4 \ln\left|\dot{\varepsilon}^*\right|\right]\left[1 + D_5 T^*\right]$$

Pressure dependence — Strain rate dependence — Temperature dependence

Ductile Failures on low deformable materials (e.g. Inermet 180)

Plastic Strain Failure

$$\begin{cases} \varepsilon_{Pl} \geq \varepsilon_{Pl}^{MAX} \\ D = 1 \end{cases}$$

Ductile-Brittle Failures with very high strain rates (e.g. Shockwave reflection)

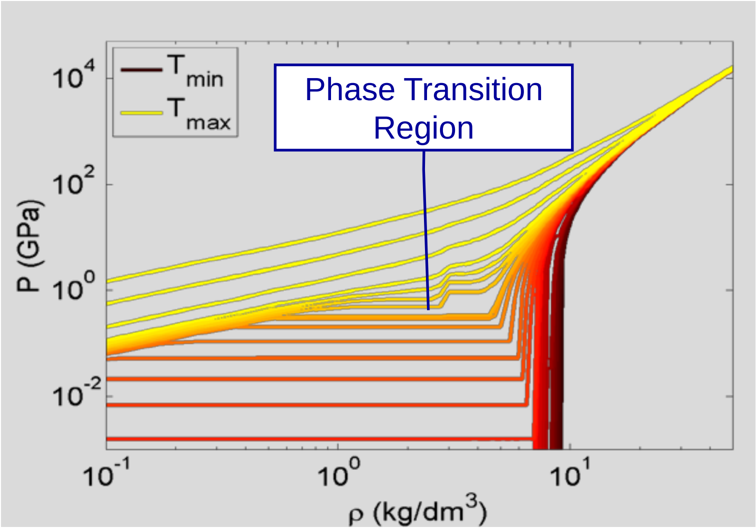
Spallation (Hydrostatic Tensile Failure)

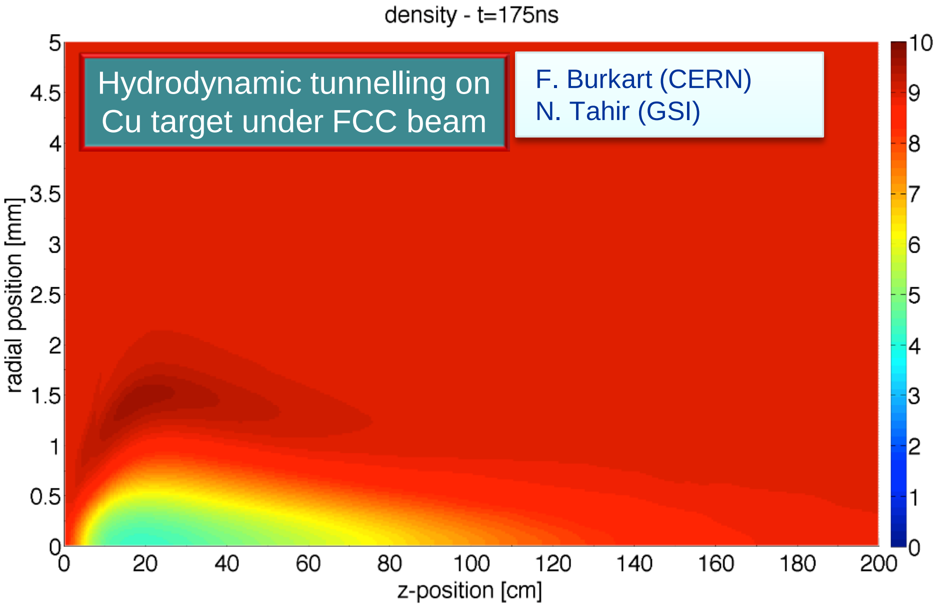
$$\begin{cases} P \leq P^{\min} \\ D = 1 \end{cases}$$

D (Damage): when D=1 → Element Failure

Hydrodynamic Tunneling

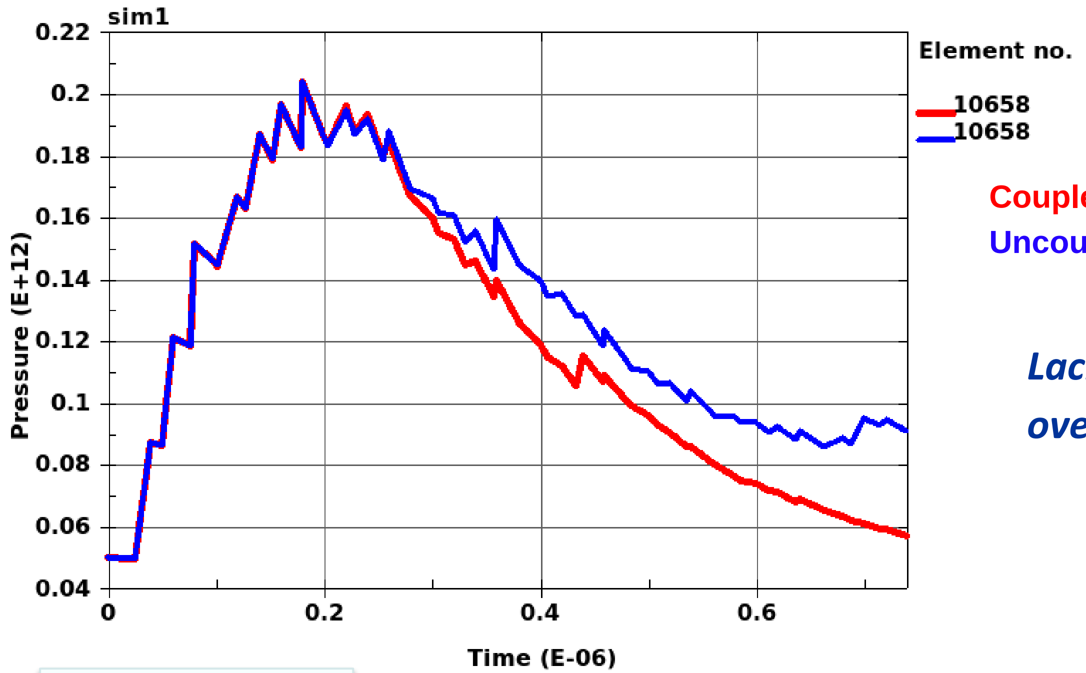
- Assume a highly energetic beam impacts a cylindrical target on its axis.
- **Temperature** and **pressure** are **dramatically increased** in the beam **interaction region.**
- Material **density** at target core is significantly **reduced** by two concurring effects:
  - Upon **phase transitions** density abruptly changes (prevailing effect).
  - **Intense shockwaves** are generated and propagate radially, displacing material outwards hence affecting its density
- If phenomena fully develop while the impact is on-going, subsequent bunches interact with lower density material and penetrate deeper into the material (**Hydrodynamic Tunneling**).





Hydrodynamic Tunneling: Code coupling

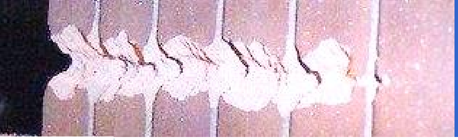
- A correct assessment of tunneling and similar effects requires a **coupling between Montecarlo interaction code and Thermomechanical code**.
- **Energy deposition map must be recalculated** each time density changes exceed a minimum threshold (typically a few percent).
- *E.g. for impacts on a W target, 7 TeV energy, 25 ns bunch spacing, coupling is required when the number of bunches is higher than 10*



Coupled
Uncoupled

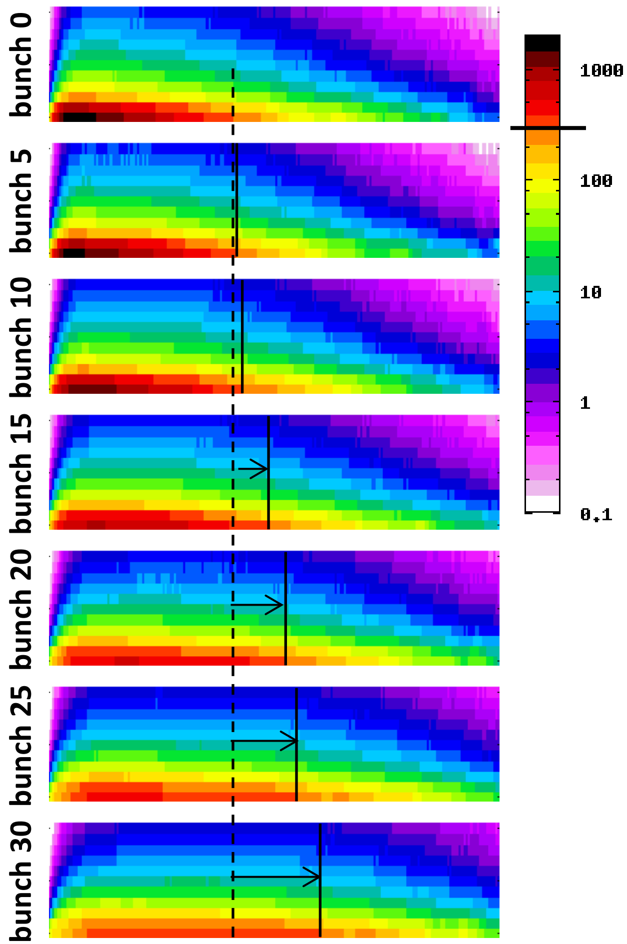
Lack of code coupling leads to overestimation of the pressure

L. Peroni, M. Scapin (Politecnico di Torino)



Hydrodynamic Tunneling: Code coupling

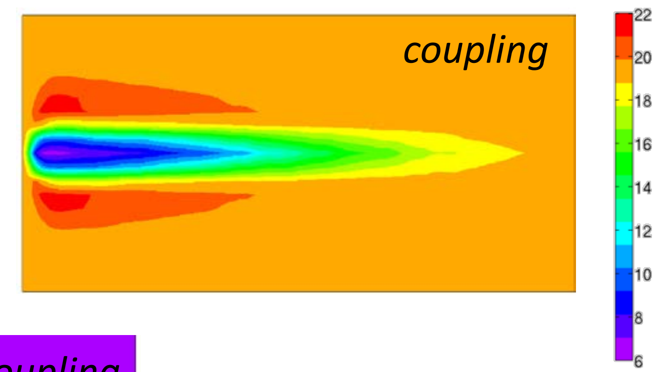
Tungsten target impacted by a train of LHC bunches (7 TeV).



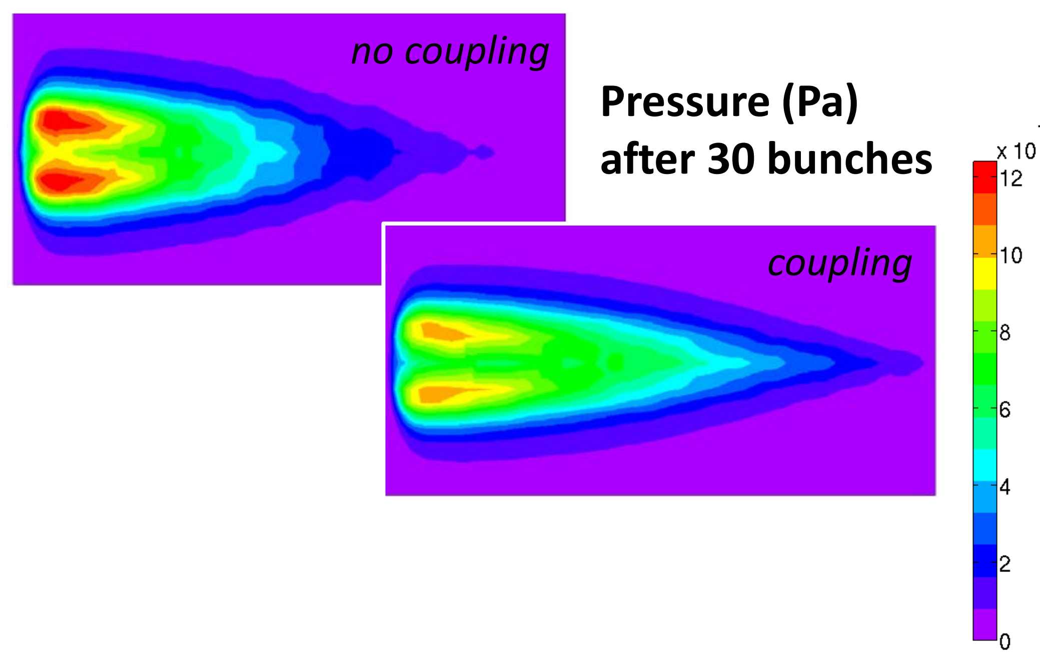
Energy deposition GeV/cm^3

L. Peroni, M. Scapin (PoliTo)

Density (g/cm^3) after 30 bunches



Pressure (Pa) after 30 bunches



End of Lecture 1

Quasistatic Stresses in Cylinders and Disks

- Transversal energy deposition profiles can often be approximated with a **Normal Gaussian Distribution**. Hence, initial temperature field in a disk or circular cylinder takes the form:

$$T(r,\tau) = T_0(r) = T_{\max} e^{-\frac{r^2}{2\sigma^2}}$$

where $\boldsymbol{\sigma}$ *is the* ***standard deviation*** *of the normal distribution*

- For a **long cylinder**, the solution at time at the end of the impact ($\boldsymbol{t = \tau}$) is given by:

$$\sigma_r'(r,\tau) = \frac{E\alpha T_{\max}}{1-\nu}\left[\frac{\sigma_b^{\,2}}{R^2}\left(1-e^{-\frac{R^2}{2\sigma_b^{\,2}}}\right) - \frac{\sigma_b^{\,2}}{r^2}\left(1-e^{-\frac{r^2}{2\sigma_b^{\,2}}}\right)\right] \qquad (7a)$$

$$\sigma_\theta'(r,\tau) = \frac{E\alpha T_{\max}}{1-\nu}\left[\frac{\sigma_b^{\,2}}{R^2}\left(1-e^{-\frac{R^2}{2\sigma^2}}\right) + \frac{\sigma_b^{\,2}}{r^2}\left(1-e^{-\frac{r^2}{2\sigma_b^{\,2}}}\right) - e^{-\frac{r^2}{2\sigma_b^{\,2}}}\right] \qquad (7b)$$

- **Homework**: Calculate the initial stresses for the case of a Rectangular Temperature Distribution given by $\boldsymbol{T(r,\tau) = T_0(r) = T_{max} H(d-r)}$ where $\boldsymbol{H(d-r)}$ is the Heaviside step function centered at $\boldsymbol{r = d}$ and $\boldsymbol{d = \sqrt{2\sigma}}$ is chosen to ensure that the total energy is the same in two cases.