Beam Induced Damage Mechanisms and Their Calculation

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- Part I: Introduction to Beam-induced Accidents
- Part II: Analysis of Beam Interaction with Matter
- Part III: Design Principles of Beam Interacting Devices
- Part IV: Experimental Testing and Validation



Objectives and Scope of the Lectures

- We deal with rapid and intense interactions between particle beams and accelerator components (typically lasting ns to μs). We do not treat here other energy release mechanisms (e.g. of stored magnetic energy)
- Focus on damage mechanisms occurring in the μs scale. Longer term phenomena
 (e.g. radiation damage) are not extensively covered
- Mainly treat components directly exposed to interaction with beam (Beam Interacting Devices)
- However, **mechanisms extend to any other component** accidentally and rapidly interacting with energetic beams (vacuum chambers, magnets, cavities).
- Mostly treating **isotropic materials**. Principles can be extended to **anistropic** materials with some mathematical complexity
- In <u>first lecture</u>, focus is given on the theoretical and thermo-mechanical principles allowing to **analyze the phenomena**.
- In <u>second lecture</u>, we deal with the **design of beam interacting systems** treating aspects as figures of merit, intensity limits, advanced materials, testing facilities etc.





- Part I: Introduction to Beam-induced Damage
- Part II: Analysis of Beam Interaction with Matter
- Part III: Design Principles of Beam Interacting Devices (BID)
 - Introduction to Failure Criteria
 - Material Selection: Figures of Merit
 - Materials for Beam Interacting Devices
- Part IV: Experimental Testing and Validation



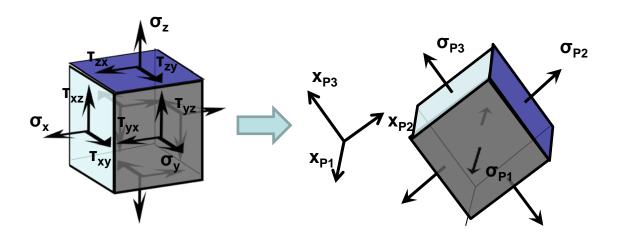


- ✓ General State of Stress
- ✓ Failure Criteria: von Mises
- ✓ Failure Criteria: Stassi d'Alia
- ✓ Deformation to Failure



General State of Stress – Failure Criteria

- Failure Theories were developed (mostly empirically) to predict failure in case of combined state of stress
- Many theories are based on the reduction of the complete 3D stress state to one in which only normal stress acts along each of the 3 principal directions.

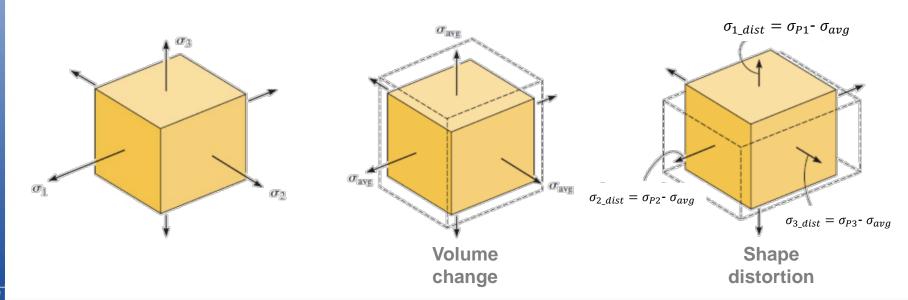


- No single Failure Theory is suitable to every material under any state of stress and for all conditions!
- Safety coefficients are adopted to protect against the approximation of Failure Criteria and the uncertainties in the state of stress knowledge.



Maximum-distortion-energy theory (or von Mises-Huber Yield criterion)

- Suitable for Ductile Materials. Extensively used.
- Total Strain Energy can be considered as the sum of two parts, one representing
 the energy causing volume change with no change in shape, and the other
 representing the energy distorting the element.
- Failure (by plastic yielding) is assumed to occur when the Distortion Energy in the material reaches the same critical value as in a tension test at yielding.





Failure criteria: von Mises and Stassi – d'Alia

Maximum-distortion-energy theory (or Von Mises-Huber Yield criterion)

Resistance condition

$$\sigma_{Y} \geq \sigma_{EQ} = \frac{1}{\sqrt{2}} \cdot \sqrt{(\sigma_{P1} - \sigma_{P2})^{2} + (\sigma_{P2} - \sigma_{P3})^{2} + (\sigma_{P3} - \sigma_{P3})^{2$$

Pressure-modified von Mia

elastic, elasto-plastic, shock), depending on material properties For baseline conditions, seek elastic regime!! and loading conditions

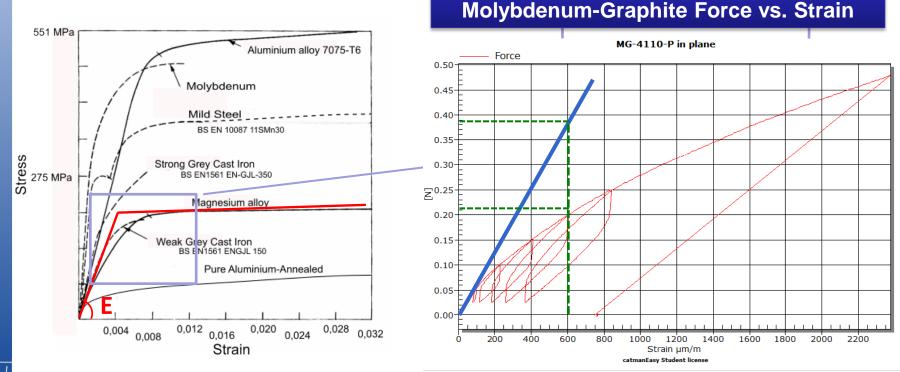
$$k = -rac{\sigma_{ ext{strength _compressio n}}}{\sigma_{ ext{strength _tension}}}$$

$$P = -\frac{\sigma_{P1} + \sigma_{P2} + \sigma_{P3}}{3}$$



Deformation to Failure

- The linear approximation is a powerful mean to describe the stress-strain relationship.
- For some materials, though, σ-ε can depart appreciably from linearity. Examples are:
 Copper, Aluminum and Magnesium alloys, and particularly Graphitic materials ...
- For deformation-driven problems (e.g. beam-induced energy deposition),
 overestimation can be made when considering tension as the limiting factor
- Deformation to Failure is a more realistic criterion is such cases!!







- ✓ General Recommendations for Materials
- ✓ Figures of Merit: Thermomechanical Robustness
- ✓ Figures of Merit: Thermal Stability
- ✓ Figures of Merit: Electrical Conductivity
- ✓ Figures of Merit: Radiation Resistance



General Recommendations for Materials

- The choice of a particular material for a BID, as much as for any other system, is driven by the material performance under different points of view
- Such aspects may be general for all applications or component-driven
 Some component-driven requirements include ...
 Radiation Hardness, Robustness, UHV Compatibility, Industrial feasibility of large components, Possibility to machine, braze, join, coat ..., Cost ...

Q: How to decide amongst a number of materials in the early phase of design?

 Relevant parameters can be turned into a set of arbitrary Figures of Merit (FOMs), allowing to rank materials against a specific requirement

IMPORTANT! Figures of Merit rely on simplified, constant, temperature-independent material properties.
They should be used as indicative, comparative tools in the design phase and not for quantitative assessment of performance!







Figures of Merit: Thermomechanical Robustness

Thermomechanical Robustness Index (TRI)

$$TRI = \frac{\mathcal{E}_{Adm}}{\mathcal{E}_{Re \, al}} \left(\frac{T_m}{\Delta T_q} - 1 \right)^m$$

$$\varepsilon_{\mathrm{Re}\,al} = \overline{\alpha} \cdot \Delta T_q$$

$$\varepsilon_{Adm} = \frac{R_{M}}{\overline{E} \cdot (1 - v)}$$

$$\Delta T_q = \frac{C_R \rho^n}{c_p X_g} \propto \frac{q_d}{c_p}$$

- TRI is related to the ability of a material to withstand the impact of a short particle pulse
- In thermal shock problems, **admissible strain** is the most meaningful quantity as the phenomenon is governed by thermal deformation
- On the other hand, effective **strength** values (R_M) are much easier to obtain in literature
- The term in T_m (melting temperature) provides an indication of the loss of strength at increasing temperature
- ΔT_q is a temperature increment related to the energy deposited q_d in the material by a given particle pulse.
- Deposited energy is to some extent related to the Geometric Radiation Length X_q and material density ρ
- C_R , n, m are **arbitrary** coefficients defining the influence of various parameters.

$$TRI = \frac{R_{M}c_{p}X_{g}}{\overline{E}(1-\nu)\overline{\alpha}C_{R}\rho^{n}} \cdot (\frac{T_{m}c_{p}X_{g}}{C_{R}\rho^{n}} - 1)^{m}$$

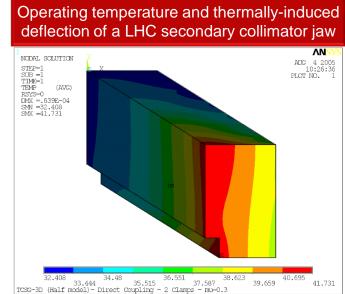


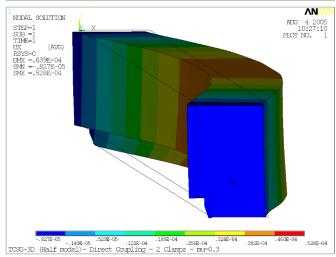
Figures of Merit: Thermal Stability

Thermal Stability Index (TSI)

- Under steady-state or slowly transient heat deposition,
 TSI provides an index of the ability of the material to maintain geometrical stability of the component.
- It is related to the inverse of the curvature of a long structure induced by a non uniform temperature distribution (for given steady-state particle losses).
- TSI is proportional to thermal conductivity and radiation length; inversely proportional to CTE and density ...
- For anisotropic materials (e.g. Carbon-Carbon, MoGr)
 weighted average properties are assumed.

$$TSI = \frac{\overline{\lambda}X_g}{\overline{\alpha}C_S\rho^n}$$



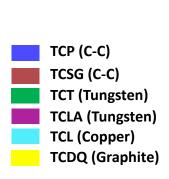


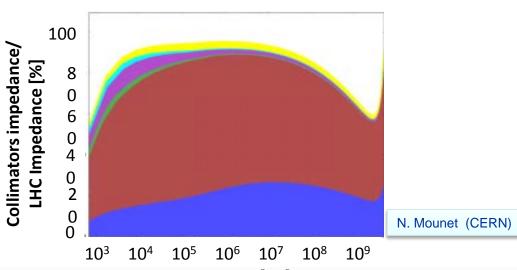


Figures of Merit: Electrical Conductivity

Electrical Condutivity (γ)

- Components located in accelerator rings (collimators, absorbers, spoilers ...) are required to minimize their contributions to **RF impedance** to limit **adverse electromagnetic effects on beam stability**.
- In "classical" regime, RF-impedance drastically increases when beam approaches the "resistive wall" ($\propto 1/b^3$) \Rightarrow contributions to impedance are much larger from components sitting close to the circulating beam as BIDs.
- RF-impedance is inversely proportional to electrical conductivity ⇒ highest electrical conductivity is sought for materials sitting closest to circulating beams!





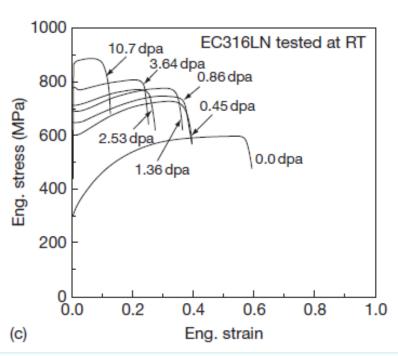


Frequency [Hz]

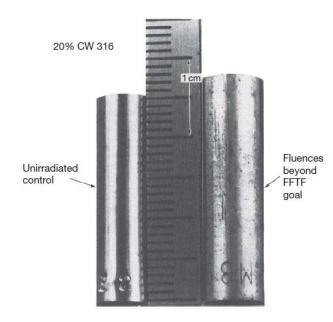
Figures of Merit: Radiation Hardness

- Irradiation of materials by energetic particles causes microstructural defects (see N. Mokhov lecture) which translate into macrostructural changes in material properties
- Many of the affected properties directly influence performance
- Such often-radical changes shall be taken into account in the design phase

Embrittlement of 316LN at different dpa levels



'Comprehensive Nuclear Materials', Editor-in-Chief: Rudy J.M. Konings, Elsevier



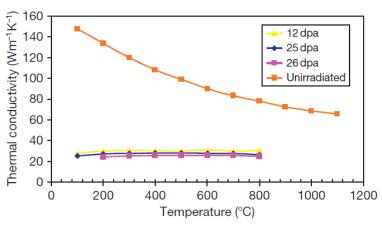
Swelling of 316 pipe after 75dpa irradiation: +33% in volume

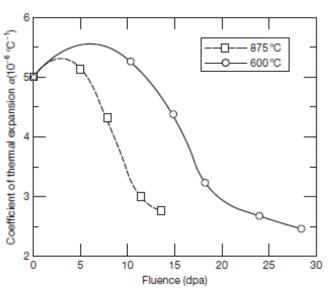


Figures of Merit: Radiation Hardness

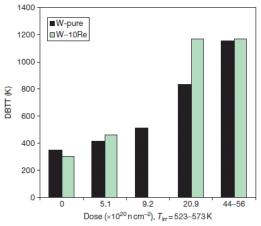
Some more examples for material of interest...

Thermal properties of nuclear grade Graphite



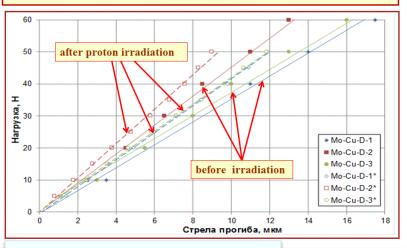


Ductile to Brittle Transition Temperature for Tungsten



Embrittlement of MoCuCD Composite

Deformation curves for irradiated (*) by 30 MeV protons and <u>unirradiated</u> Mo-Diamond samples for LHC collimator materials



A. Ryazanov (RRC Kurchatov Institute)





- ✓ Material Requirements
- ✓ Novel Materials R&D Program
- ✓ Novel Materials: Copper-Diamond
- ✓ Novel Materials: Molybdenum-Graphite
- ✓ Material Comparison



Material Requirements

As seen, maximizing FOMs requires the optimization of a number of material properties ...

- Maximize **Electrical Conductivity** (γ) to limit RF Impedance
- Maximize Thermal Conductivity (λ) to maintain geometric steady-state losses (TSI)
- Difficult to combine all requirements in one existing Widespread, on-going efforts in developing novel etrical Minimize CTE (α) to increase resistance stability (TRI and TSI)
- Maximize Melting/Degradation gh temperatures reached in case of bear
- Maximize Specific crease during impacts (TRI)
- Maximiza Tarly strain to failure) to improve thermal shock
- number (**Z**) to limit peak energy deposition while Balance maintain
- Minimize R **induced Damage** to improve component lifetime under long term particle irrad

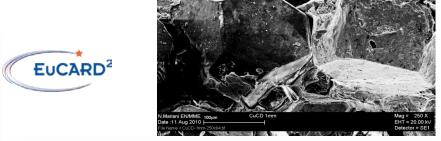


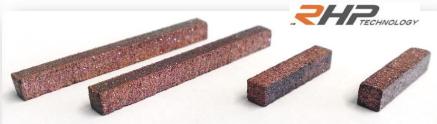
Novel Materials R&D Program

- Extensive R&D program launched at CERN in partnership with industries and other institutions.
- Aim: explore/develop **composites** combining the properties of **graphite** or **diamond** (low ρ , high λ , low α) with those of **metals** and **transition metal-based ceramics** (high R_M , good γ).
- Amongst many investigated materials, most interesting are Copper-Diamond and particularly Molybdenum Carbide-Graphite.

 Production techniques include Rapid Hot Pressing, Liquid Phase Sintering and Liquid Infiltration.







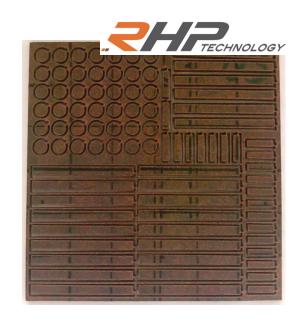


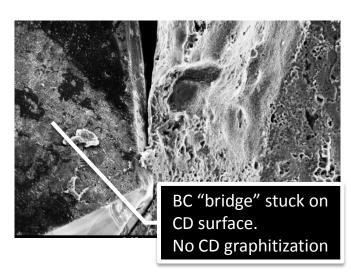
Novel Materials: Copper-Diamond

Developed by RHP-Technology (Austria)

Composition:

- 60%v diamonds (90% 100 μm, 10% 45 μm)
- 39%v Cu powder (45 μm)
- 1%v B powder (5 μm)
- No diamond degradation
- Thermal (~490 Wm⁻¹K⁻¹) and electrical conductivity (~12.6 MSm⁻¹)
- No direct interface between Cu and CD (lack of affinity). Partial bonding bridging assured by Boron Carbides limits mechanical strength (~120 MPa).
 - Cu low melting point (1083 °C)
 - CTE increases significantly with T due to high Cu content (from ~6 ppmK⁻¹ at RT up to ~12 ppmK⁻¹ at 900 °C)







N

Novel Materials: Molybdenum-Graphite

- Co-developed by CERN and Brevetti Bizz (Italy)
- Broad range of processes and compositions investigated (Molybdenum, Natural Graphite, Mesophase pitch-based Carbon Fibers).

Why Molybdenum?

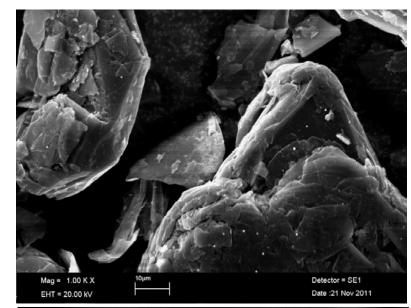
- Refractory metal
- Density lower than Tungsten

Why Natural Graphite?

- Low CTE (along basal plane)
- High Thermal Conductivity (along basal plane)
- Low Density
- Very High Service Temperatures
- High Shockwave Damping
- Low cost

Why Mesophase Pitch-based Carbon Fibres?

- Increase mechanical strength
- Contribute to Thermal Conductivity (highly ordered structure)

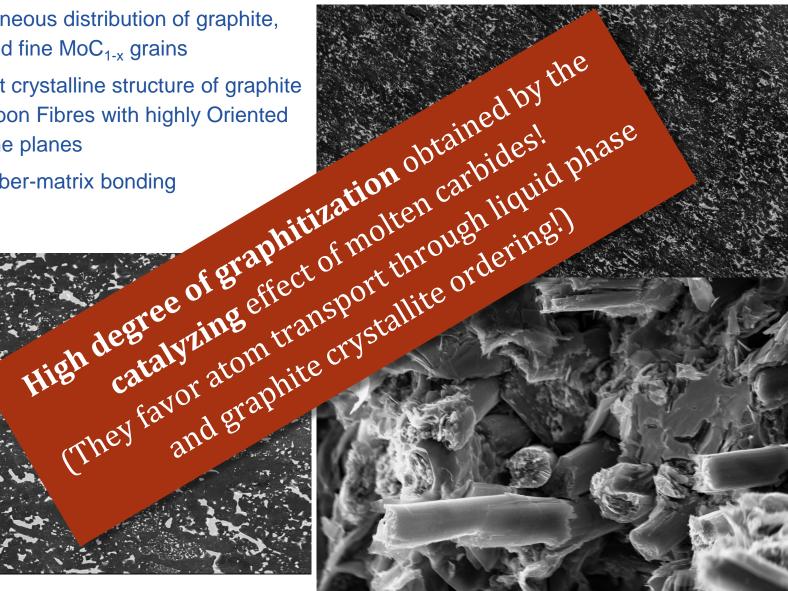




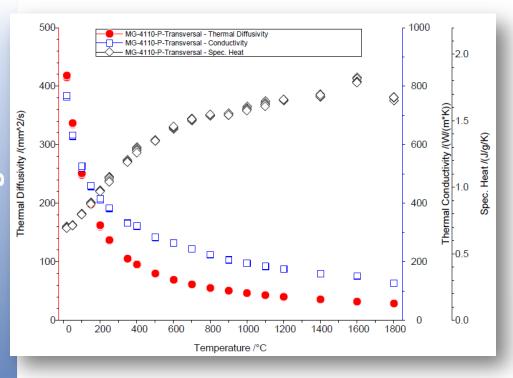


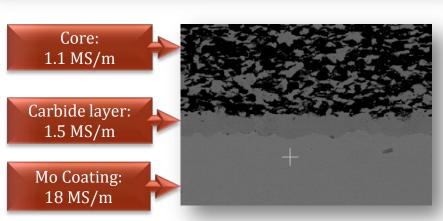
Novel Materials: Molybdenum-Graphite

- Homogeneous distribution of graphite, fibers and fine MoC_{1-x} grains
- Excellent crystalline structure of graphite and Carbon Fibres with highly Oriented Graphene planes
- Strong fiber-matrix bonding

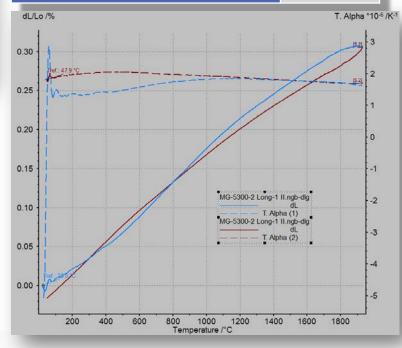


Molybdenum-Graphite Properties





ρ [g/cm³]	2.5
$lpha_{\perp}$ (RT to 1000° C) [10 ⁻⁶ K ⁻¹]	<1.8
$lpha_{/\!/}$ (RT to 1000° C) [10 ⁻⁶ K ⁻¹]	12
$λ_{\perp}$ (RT) [W/mK]	>770
λ _{//} (RT) [W/mK]	85
$σ_{\perp}$ (RT) [MS/m]	1÷18
σ _{//} (RT) [MS/m]	0.3
E (Flexural) [GPa]	53
R _{FI} [MPa]	85





FOMs: Material Comparison

Material	Beryllium	Carbon- Carbon	Graphite	Molybdenum Graphite	Copper- Diamond	Glidcop ®	Molybdenum	Tungsten Alloy (IT180)
$ ho$ [g/cm 3]	1.84	1.65	1.9	2.50	5.4	8.90	10.22	18
Z	4	6	6	~6.5	~11.4	~29	42	~70.8
X _g [cm]	35	26	19	17	4.8	1.4	0.96	0.35
$oldsymbol{c}_p$ [Jkg ⁻¹ K ⁻¹]	1925	780	760	750	420	391	251	150
ᾱ [10 ⁻⁶ K ¹]	18.4	4.1	5.5	5.0	7.8	20.5	5.3	6.8
λ̄ [Wm ⁻¹ K ⁻¹]	216	167	70	547	490	365	138	90.5
T _m [°C]	1273	3650	3650	2589	~1083	1083	2623	~1400
\overline{E} [GPa]	303	62.5	12	44	220	130	330	360
R _M [MPa]	370	87	30	80	70	365	660	660
ΔT_{q} [K]	0.36	1.2	1.7	2.1	15.1	60.1	144	745
TRI [–]	790	1237	1101	634	6.8	5.3	6.4	0.5
TSI [-]	17.1	44.6	10.1	69.4	9.9	0.8	0.7	0.1
γ [MSm ⁻¹]	23.3	~0.14	~0.07	~1÷18	~12.6	53.8	19.2	8.6



FOMs: Material Comparison

- The higher the FOM, the better the material ... No one-fits-it-all material!
- Carbon-based materials feature excellent TRI and TSI thanks to low-Z, low CTE, low density, high degradation temperature, high conductivity
- **Beryllium** is outstanding under practically all points of view ... unfortunately its used is severely limited by its **toxicity**.
- However low electrical conductivity penalizes C-C and graphite if RF-impedance is an
 issue. In such a case, MoGr is the most promising compromise, particularly if coated
 with higher conductivity thin films.
- Note poor performance of Tungsten Alloy, also due to the low melting temperature of the Ni-Cu matrix required to reduce material brittleness ... it is not pure W!

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- Part I: Introduction to Beam-induced Damage
- Part II: Analysis of Beam Interaction with Matter
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- Part IV: Experimental Testing and Validation
 - Why Experimental Tests?
 - HiRadMat Facility
 - HiRadMat Experiments



Why Experimental Tests?

Why is experimental validation important?

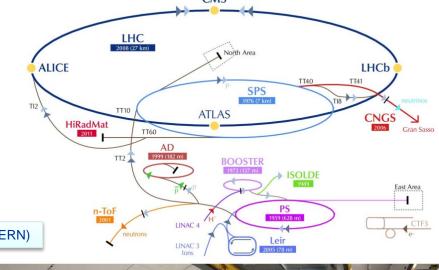
- With accidental beam impacts, one enters a relatively unknown territory, that of **high** power explosions and ballistics.
- When large density changes, phase transitions, fragmentations are involved, one has to resort to special advanced tools (Hydrocodes).
- These state-of-the-art wave propagation codes can be very reliable, provided the complex material models required are available and precise.
- Existing material constitutive models at extreme conditions are limited and mostly drawn from military research (classified). They are often unavailable for specific alloys and composites.
- Additional consequences on UHV, electronics, bellows cannot be easily anticipated by numerical simulations.
- Only ad-hoc material tests can provide the correct inputs for numerical analyses and validate/benchmark simulation results on simple specimens as well as on complex structures.
- A dedicated facility has been designed and commissioned at CERN to test materials and systems under high intensity pulsed particle beams: HiRadMat (High Radiation to Materials).

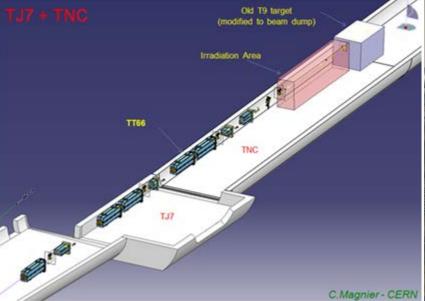


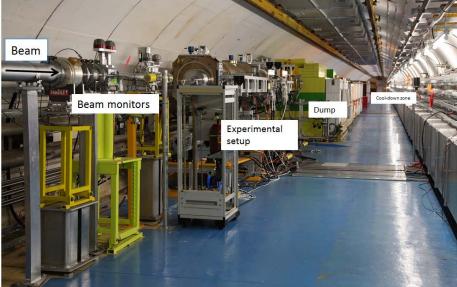
HiRadMat (High Radiation to Materials) Facility

- Dedicated facility for studying the impact of intense pulsed beams on materials
 - Material damage
 - Material vaporization
 - Thermal management
 - Radiation damage to materials
 - Beam-induced pressure waves
 - 9 experiments in 2012

A. Fabich, I. Efthymiopoulos (CERN)









HRMT12 Experiment

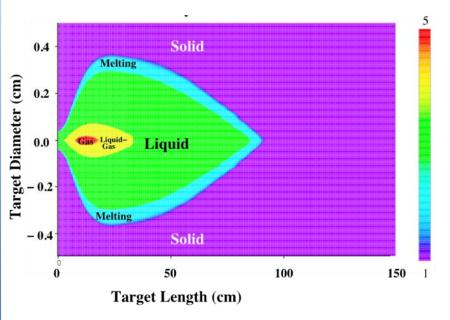
Experiment Goals

- Show damage of SPS beam impacting on target.
- Benchmarking of hydrodynamic tunneling simulations.
- **Target**: Copper, 3 x 15 blocks, length 10cm and radius 4cm.
- Experiment with SPS beam in HiRadMat.
- 440GeV/c.
- 108 or 144 bunches with 1.5e11 p per bunch.
- Bunch trains of 36 bunches.
- Bunch spacing 50ns.
- Beam size $\sigma = 0.2$ or 2mm.

J. Blanco, R. Schmidt *et al* (CERN)



Comparison test results vs. simulations



Third target, 144 bunches delivered after 7.85 μ s To provide the second of the sec

Target Length (cm)





N. Tahir (GSI)

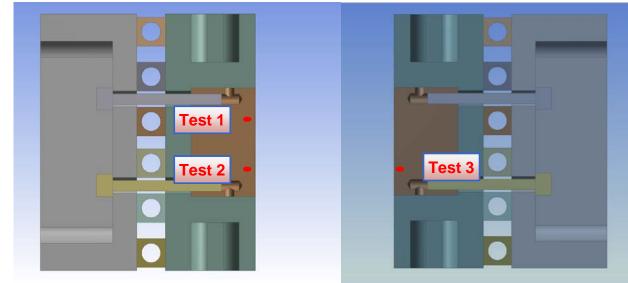
R. Schmidt et al (CERN)

50 60 70 80 90 100 110 120 130 140 150

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Experiment Goals

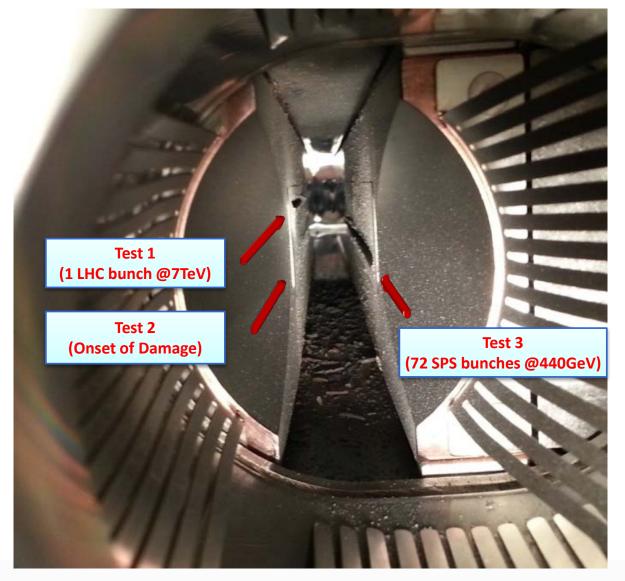
- Integral test under SPS beam of 2 LHC Tertiary Collimator Jaws
- Beam energy: 440 GeV
- Impact depth:2mm



	Test 1	Test 2	Test 3	
Goal	Beam impact equivalent to 1 LHC bunch @ 7TeV	Identify onset of plastic damage	Induce severe damage on the collimator jaw	
Impact location	Left jaw, up (+10 mm)	Left jaw, down (-8.3 mm)	Right jaw, down (-8.3 mm)	
Pulse intensity [p]	3.36×10^{12}	1.04×10^{12}	9.34×10^{12}	
Number of bunches	24	6	72	
Bunch spacing [ns]	50	50	50	
Beam size [σ _x - σ _v mm]	0.53 x 0.36	0.53 x 0.36	0.53 x 0.36	



Post-irradiation visual inspection



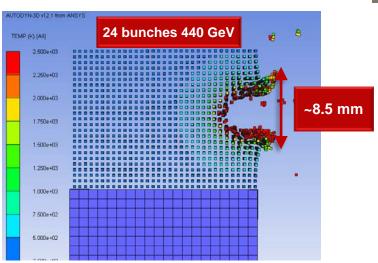


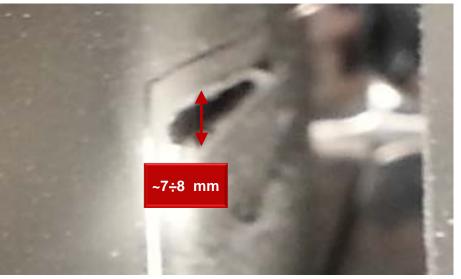
State Miles

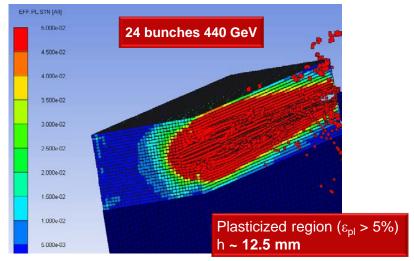
HRMT09 Experiment

Analysis of Test 1

- Goal: beam impact equivalent to 1 LHC bunch @ 7TeV
- Intensity 1.5 x 10¹¹p
- Qualitative damage evaluation
- Groove height ~ 7 mm,
 in good agreement with simulations









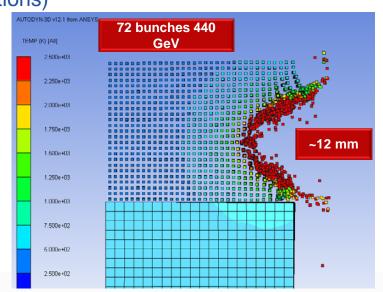
HRMT09 Experiment

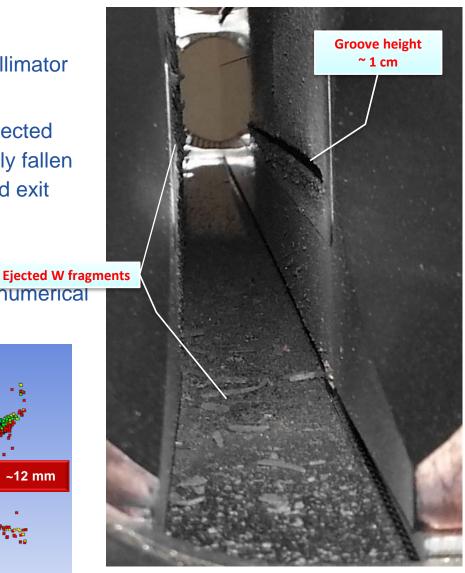
Analysis of Test 3

- Goal: induce severe damage on the collimator (~3 equivalent LHC bunches)
- Impressive quantity of tungsten alloy ejected (partly bonded to the opposite jaw, partly fallen on tank bottom or towards entrance and exit flanges)

Vacuum degraded. Tank contaminated

Groove height ~ 1 cm (consistent with numerical simulations)







Experiment Goals

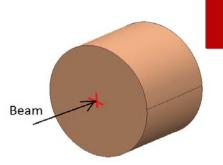
- Benchmark advanced numerical simulations and material constitutive models through extensive acquisition system
- Characterize six existing and novel materials currently under development for future Collimators: Inermet180, Molybdenum, Glidcop, MoCuCD, CuCD, MoGr

• Collect, mostly in **real time**, experimental data from different acquisition systems (Strain Gauges, Laser Doppler Vibrometer, High Speed video Camera,

Temperature and Vacuum probes)

Beam Parameters					
440 GeV					
1.1e11					
25 ns					

High Intensity Tests: Sample: half-moon; Beam Offset 2 mm



Beam

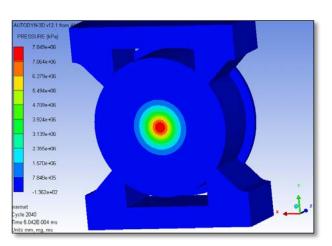
Medium Intensity Tests:

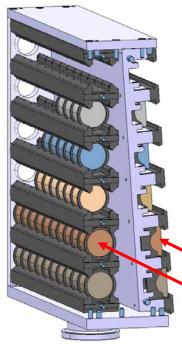
Sample: Ø 40 mm, L30 mm



Medium Intensity Beam Impacts:

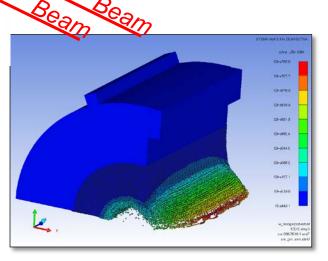
- Hoop strain measurements on the surface of the sample;
- Radial vibration measurements;
- Temperature measurements;
- Sound measurements.





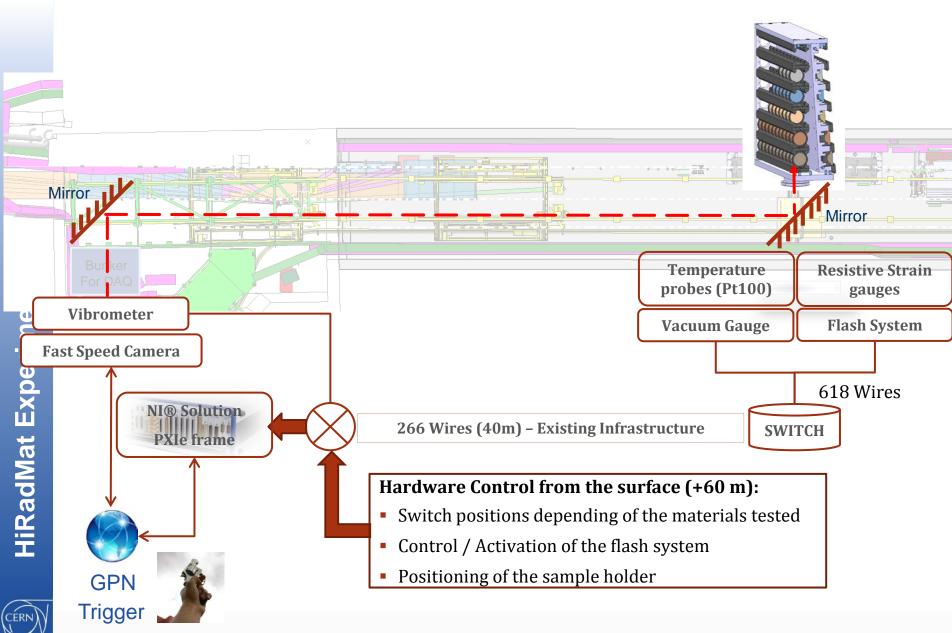
High Intensity Beam Impacts:

- Hoop strain measurements on the surface of the sample;
- High-speed camera to follow the fragment front formation and propagation;
- Temperature measurements;
- Sound measurements.



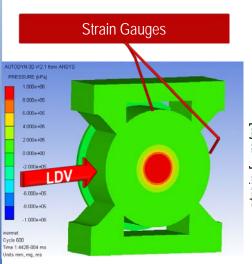


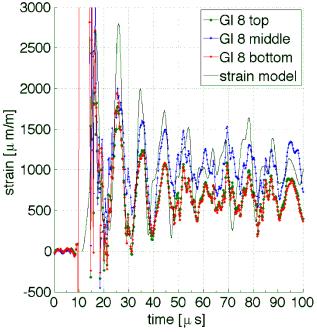
HRMT14 Experiment

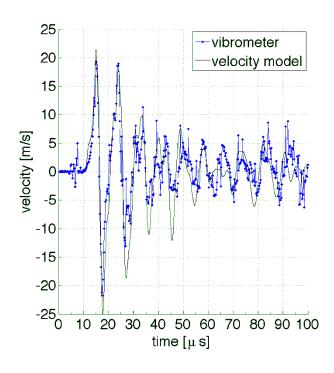


Medium Intensity Tests

- Extensive hydrocode numerical analysis (Autodyn).
- Comparison of simulated circumferential strains and radial velocity with measured values on sample outer surface.



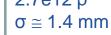




Inermet180 24 b (scraped)

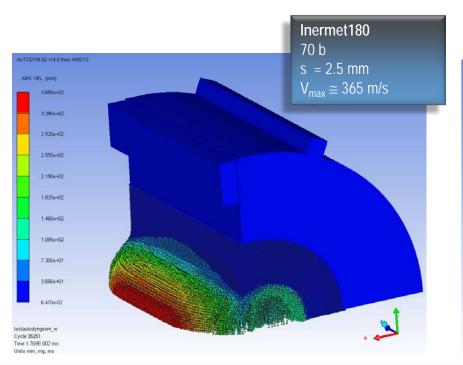
Total intensity:

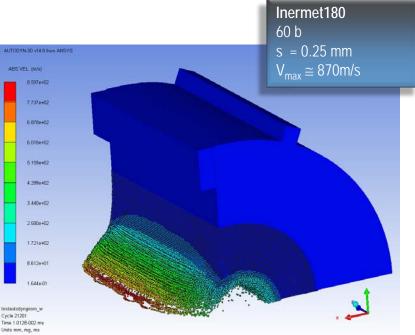
2.7e12 p



High Intensity Tests

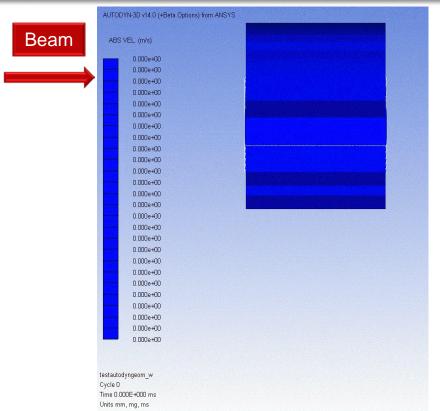
- Smooth-Particle-Hydrodynamics (SPH) calculations allowed determining damage extension, particle fragment velocity and trajectories.
- Assessment of potential damages to tank, windows and viewports.
- Material density changes.

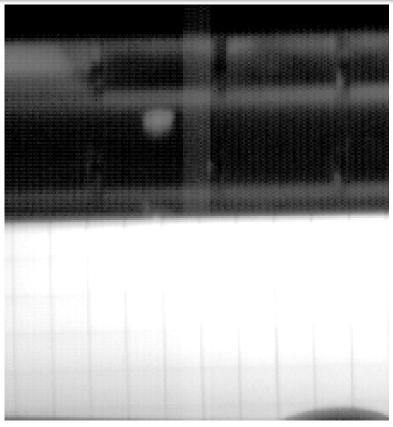






High Intensity Tests: Comparison between numerical simulation (SPH) and experiment

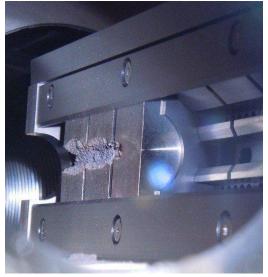




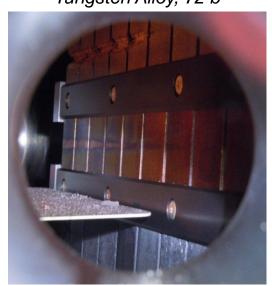
Case	Bunches	p/bunch	Total Intensity	Beam Sigma	Specimen Slot	Velocity
Simulation	60	1.5e11	9.0e12 p	2.5 mm	9	316 m/s
Experiment	72	1.26e11	9.0e12 p	1.9 mm	8 (partly 9)	~275 m/s







Tungsten Alloy, 72 b



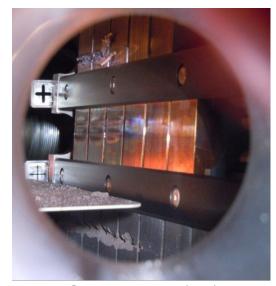
Copper-Diamond 144 b



Molybdenum, 72 & 144 b



Molybdenum-Copper-Diamond 144 b

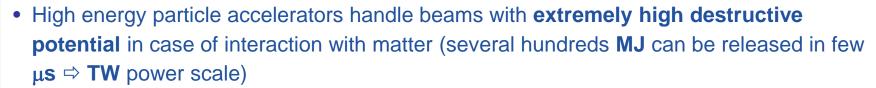


Glidcop, 72 b (2 x)



Molybdenum-Graphite (3 grades) 144 b





- The analysis of beam-matter interaction involves several disciplines and requires a multiphysics approach
- When interaction phenomena do not lead to extensive changes of density or phase transitions, material response can be analysed with a good degree of approximation by thermoelasticity principles
- Otherwise, advanced nonlinear tools (hydrocodes) must be invoked: these numerical codes rely on complex material constitutive models encompassing the full range of states of matter
- A number of indicative **Figures of Merit** can help in the material selection process in the early design phase of systems exposed to beam interaction
- No material fits all requirements! However, a new generation of metal- and ceramic-matrix composites with diamond or carbon reinforcements is showing promising results, in particular Molybdenum Carbide Graphite
- Only dedicated, carefully designed experiments in ad-hoc facilities (e.g. HiRadMat) can benchmark advanced numerical simulations and provide the final validation for systems potentially exposed to interaction with highly energetic beams







Homework Problem

A L=100 mm long target rod with circular cross-section of radius R = 2.5 mm made of isotropic graphite is impacted at its center by a train of $n_b = 288$ proton bunches. Each bunch has a population of $n_p = 1.5 \times 10^{11}$ protons. Bunches are separated by $t_b = 25$ ns.

The following material properties are uniform and can be assumed temperature-independent: density $\rho = 1.85$ g/cm³, thermal conductivity $\lambda = 70$ W/mK, CTE $\alpha = 4 \times 10^{-6}$ K⁻¹, Young's Modulus E = 10.5 GPa, Poisson's ratio v = 0.15. Assuming that the energy deposition profile is **uniform** in the longitudinal direction, with an **axially symmetric** Gaussian distribution (standard deviation **0.6** mm) and a peak deposited energy density per proton of $q_p = 2.46 \times 10^{-11}$ J/g, do the following:

- 1. Calculate the peak energy density $\mathbf{q}_{\mathbf{dMax}}$ and peak power density $\dot{\mathbf{q}}_{\mathbf{dMax}}$ at the target center deposited during the impact.
- 2. Write the distribution of the energy $\mathbf{q_d}(\mathbf{r})$ deposited on the target cross-section during the impact. Calculate the total deposited energy per unit length $\mathbf{Q_d}$.
- 3. Assume a reasonable average value for the specific heat $\mathbf{c_p}$, whose evolution with temperature is given in the plot below. Justify your choice.
- 4. Determine the thermal diffusion time and verify if the heat deposition can be considered "instantaneous".
- 5. Based on your conclusions on previous question, determine the initial temperature distribution on the cross section, its maximum value and its final uniform value (assuming an adiabatic problem)



Homework Problem

- 6. Assuming that the rod is restrained at both its ends, determine, at the time of maximum stresses, the quasistatic radial, circumferential and axial stresses at the center and on the outer rim of the rod.
- 7. Given that, in reality, the rod is free to axially expand, calculate the period of axial stress waves.
- 8. Determine the maximum value of the dynamic axial stress to be superposed to the quasistatic stresses calculated at step 6.
- 9. Draw an approximate plot of the dynamic axial stress at mid-rod as a function of time. Comment on the time structure of the stress curve.
- 10. Calculate the maximum total axial stress on the outer rim.

